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THE

PHYSICAL REVIEW.

A COMPARISON OF ALTERNATING AND DIRECT SPARK POTENTIALS.

By J. C. Jensen.

THE potential required to produce a disruptive discharge across a spark gap has received the attention of many investigators. Until comparatively recent years these measurements have been chiefly concerned with direct current, the source of potential usually being a Holz or a Wimshurst machine. A fairly complete list of these contributions beginning with the work of Lord Kelvin in 1860 may be found in J. J. Thompson's "Conduction of Electricity through Gases."[1]

With the development of high-frequency oscillatory discharges such as are used in wireless telegraphy, and high tension alternating current for the transmission of power, the question of alternating spark potentials has received the attention of engineers and physicists because of the practical value attached to such information. Steinmetz[2] has measured the A.C. potential required to produce a spark in air at ordinary pressures and found a practically linear relation between potential and spark distance up to 110-$KV._{eff}$ with a commercial current of 125 cycles. Muller,[3] who used a different method, obtained similar results. Toepler[4] has collected tables from Freiberg, Jona, Steinmetz and others and attempted to reduce all to standard conditions for purposes of comparison, but with unsatisfactory results. Kowalski and Rappel[5] have made one of the most careful series of measurements and found their mean effective values for alternating potentials from 4 to 6 per cent. lower than those given by Toepler. Among others who have worked

[1] J. J. Thompson, Conduction of Elect. through Gases, p. 436.
[2] Steinmetz, Am. Inst. E. E. Trans., 15, p. 294.
[3] Muller, Ann. der Physik, 28, p. 585.
[4] Toepler, Electrotech. Zeitschr., 28, p. 1000.
[5] Kowalski and Rappel, Phil. Mag., 18, p. 699.

in this field may be mentioned Weicher,[1] Reisenfeld,[2] Peek,[3] and Watson.[4]

While the relation of direct to alternating spark potentials has both theoretical and practical value, the difficulty in getting results capable of comparison has resulted from the fact that the measurements to be compared have been made under different conditions, by different methods, and different observers. Some hold that the alternating maximum potential for a given gap equals the direct, and others believe that the mean effective is more nearly the correct value. To settle this question by measuring first, say, the direct, and then the alternating potential under identical conditions of electrode material, spark distance, temperature, gas and pressure, was the purpose of the following work.

APPARATUS.

A diagram of the apparatus used is shown in Fig. 1. The spark gap, S, is connected with a set of variable liquid resistances, R_3. Direct potential was supplied through the switch K_4, by four 1,000 volt generators connected in series, or alternating potential by an open-core transformer, Tr, excited by a 60-cycle, 220-volt power supply. The alternating potential was varied by a flexible lamp-bank resistance placed in the primary circuit of this transformer. The direct potential could be varied by the use of resistances placed in the field circuits of the generators. The current flowing through the gap at the moment of discharge was measured by a Weston A.C. voltmeter, calibrated to read in milliamperes. A high-resistance telephone receiver, P, shunted around a non-inductive resistance, R_2, served as a sensitive detector of oscillatory discharges and of the shorter sparks. Potentials were measured by a potentiometer system consisting of : $-$ R, a specially constructed, non-inductive resistance of 600,000 ohms in steps of 20,000; R', a smaller resistance; the galvanometer, G; (all of Leeds & Northrup make); and the battery, B. The contactor or Joubert disk, C, was rotated by a synchronous motor connected in multiple with the alternating current supply. When making readings for comparison this contact was set for the peak of the alternating potential wave so that the readings indicated the maximum alternating potential. When measuring direct potential the Joubert disk was short-circuited by the switch, K_3. The Battery, B, had an E.M.F. of about 6 volts, checks being made from time to time against a Weston Normal cell. With the arrangement of

[1] Weicher, Electrotech. Zeitschr., 32, p. 463.
[2] Reisenfeld, Z. S. fur Elec. Chemie, 11, p. 725.
[3] Peek, Am. Inst. E. E. Proc., 33, p. 911.
[4] Watson, Jour. Inst. E. E., 43, p. 713.

the apparatus as shown the potential across the terminals of the potentiometer, hence also across the spark gap, is given by the equation,

$$\text{P.D.} = \frac{R + R'}{R'} E,$$

where E is the E.M.F. of the battery B.

The spark gap, S, is shown in detail in Fig. 2. Spark balls of Al and Cu, having 1 cm. radius, were used for most of the experiments. These balls could be easily removed from the tube for repolishing, which was done after each set of observations. The length of the gap could be

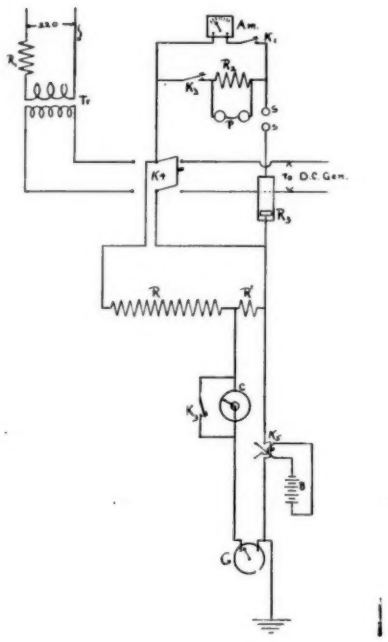

Fig. 1.

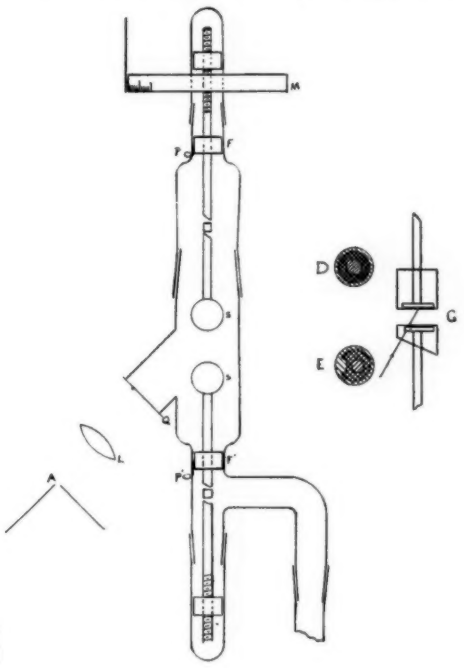

Fig. 2.

regulated by the micrometer drum, M, which had a circumference of 20 cm., making it possible to read within 1/200 of a thread on the screw, R. Platinum wires, P and P', connected respectively with the brass ferrules, F and F', through which the potential was applied to the balls. Initial ionization was furnished by the arc-lamp, A, whose ultra-violet light was focused on the upper electrode by the quartz lens, L, through the quartz window, Q. Plane electrodes of Cu and Al, constructed as shown in Fig. 2, D, E and G, were also used. The upper electrode, D (2 cm. in diameter), was covered by a piece of microscope cover glass (18 mm. in thickness), having a hole 1 cm. in diameter bored in the center.

This glass disk, which extended 2 mm. beyond the electrode on all sides, was cemented to a glass tube as shown at *G*. The glass hood was necessary in order to prevent a discharge from the edges of the electrode, and to make the field more nearly uniform. The lower electrode was similarly constructed except that a meniscus was cut from one side of it to permit of the passage of the ultra-violet light.

The gases used were air, N, H, and CO_2. These were carefully prepared and dried before using for from 12 to 24 hours in a chamber containing P_2O_5. In the case of the nitrogen (which was prepared by passing air over burning phosphorus to extract the O, and then removing any CO_2 present by leaving in contact with KOH) an unusual amount of ionization seemed present at times. This phenomenon was probably similar to that recorded by Schenck and Breuning[1] as the effect disappeared when the opening between the phosphorus tube and the drying chamber was kept closed and the gas stored for 24 hours or more before using.

MEASUREMENTS.

The experimental difficulties connected with the making of measurements were not inconsiderable. The motors which drove the direct current generators are wound for 220 volts but it was found that the available direct current power was so fluctuating that it could not be used for purposes of research; fortunately, a set of 100 lead accumulators of size sufficient to drive the motors and excite the generator fields was at hand. The campus alternating current supply was generally quite steady, yet many readings were vitiated by the untimely advent of a motor load which caused momentary fluctuations. Moreover, the unusual atmospheric humidity during the time when most of the measurements were taken caused several breakdowns in the insulation of the armatures of the D. C. generators, with subsequent expense and delay.

In making a set of measurements the plan followed was to decrease the gap length slowly with the alternating potential applied, until a spark appeared. The distance taken was that given by the average of five readings of the micrometer, or more if the first five showed any appreciable variation. The corresponding potential was then observed, potentiometer settings being made both preceding and following those on the spark length. Immediately following this, direct potential measurements were made, the micrometer setting being that of the average of the five mentioned above, and the potential being gradually increased until the spark occurred. By taking these precautions results

[1] Schenck and Breuning, Deutsch. Chem. Gesell, Ber., 47, p. 2601.

were obtained which seldom varied as much as 1 per cent. in a set of five readings.[1]

For these direct potential measurements it was found that time could be saved by using a Weston voltmeter,—the voltmeter having been first checked with the potentiometer. The relative accuracy of the measurements can be judged by inspecting Tables I., II. and III. below,—the ± magnitudes indicating the mean variation observed.

TABLE I.

Cu Spheres, 1 cm. Radius in Hydrogen. Pressure 30 cm.

Spark Lengths, Mm.	Potentials.	
	D. C.	A. C.$_{max.}$
3.175 ± .0025	2,915 ± 17.4	3,325
2.807 " .0013	2,672 " 3.6	3,070
2.413 " .0013	2,382 " 12.4	2,754
1.965 " .0013	2,060 " 6.0	2,363
1.575 " .0013	1,858 " 11.6	2,078
1.258 " .0013	1,596 " 4.4	1,763
.827 " .0013	1,245 " 2.6	1,464
.573 " .0013	1,046 " 4.0	1,133
.332 " .0013	769 " 4.0	891
.116 " .0013	538 " 3.0	661

TABLE II.

Cu Spheres, 1 cm. Radius in Hydrogen. 15 cm. Pressure.

Spark Lengths, Mm.	Potentials.	
	D. C.	A. C.$_{max.}$
6.47 ± .0025	3,078 ± 12.0	3,341
5.886 " .0050	2,842 " 18.6	3,090
5.086 " .0038	2,500 " 3.6	2,755
4.185 " .0025	2,177 " 4.4	2,375
3.41 " .0013	1,842 " 4.0	2,052
2.776 " .0013	1,586 " 6.0	1,788
2.04 " .0013	1,283 " 3.0	1,395
1.39 " .0013	1,073 " 2.0	1,150
.91 " .0000	845 " 1.0	906
.45 " .0025	588 " 2.0	649

Since the investigations of Warburg[2] it has been generally agreed that in the case of direct potentials, the chief effect of illuminating a spark gap with ultra-violet light, X-rays, or cathode rays, is to diminish the " lag," and that the spark potential is but little affected. Investi-

[1] See Kemp & Stephens, Am. Inst. E. E. Proc., Jan., 1910.

[2] Warburg, Wied. Ann., 59, p. 1; 62, p. 385.

TABLE III.

Cu Spheres, 1 cm. Radius in N. Pressure 15 cm.

Spark Lengths, mm.	Potentials.	
	D. C.	A. C.max.
2.985 ± .0013		3,352
2.970 " .0013	2,975 ± 10.2	
2.833 " .0038	2,785 " 5.4	3,095
2.338 " .0013	2,423 " 13.2	2,720
1.907 " .0025	2,105 " 5.2	2,350
1.513 " .0013	1,763 " 5.2	1,990
1.208 " .0013	1,510 " 5.6	1,696
.916 " .0013	1,232 " 8.4	1,374
.637 " .0025	967 " 3.2	1,148
.485 " .0013	746 " 3.0	894
.241 " .0013	456 " 2.0	640

gators of alternating potentials are by no means unanimous on this point however. Kowalski and Rappel[1] contend that the relatively high results obtained by Steinmetz[2] are due to the fact that the latter provided no means for producing free ions in the spark gap. Weicher[3] found that illumination decreased the spark potentials, " Anfangsspannung," by several per cent., and Moreau[4] observed a similar effect. On the other hand, Kemp and Stephens[5] find this effect negligible; and Peek[6] after going over the whole field of evidence, comes to the same conclusion so far as ordinary frequencies and pressures are concerned. In the present experiment it was found that consistent results could not be obtained in a carefully dried gas without illumination, there being variations of as much as 20 per cent. in the potential obtained from setting to setting. It was observed further that after a discharge had taken place in a darkened room another discharge immediately afterwards would take place at a lower potential, but that a longer wait would eliminate this lowering. It was also found that, with the arc off, a given potential might be applied for five minutes or more without a spark, but that the spark would at once appear when the electrodes were illuminated. To test the effectiveness of the ultra-violet light as a producer of ions, a milligram of RaBr was put into the tube as a substitute, and readings taken. Curves 3, and 4, Fig. 3, show that the results were the same in both cases, so far as regularity of discharge is concerned.

[1] *Loc. cit.*

[2] *Loc. cit.*

[3] Weicher, Electrotech, Zeitschr., 32, p. 463.

[4] Moreau, Jour. de Physique, 8, pp. 16 and 94.

[5] *Loc. cit.*

[6] Peek, Dielectric Phenomena in High-voltage Engineering, p. 198.

The difference in magnitude undoubtedly arises from other causes. It thus appears that Warburg's explanation holds equally well for both direct and alternating potentials as regards providing a source of ions.

With the potentials available, spark distances at atmospheric pressure would have been very small, hence most of the readings were taken at a

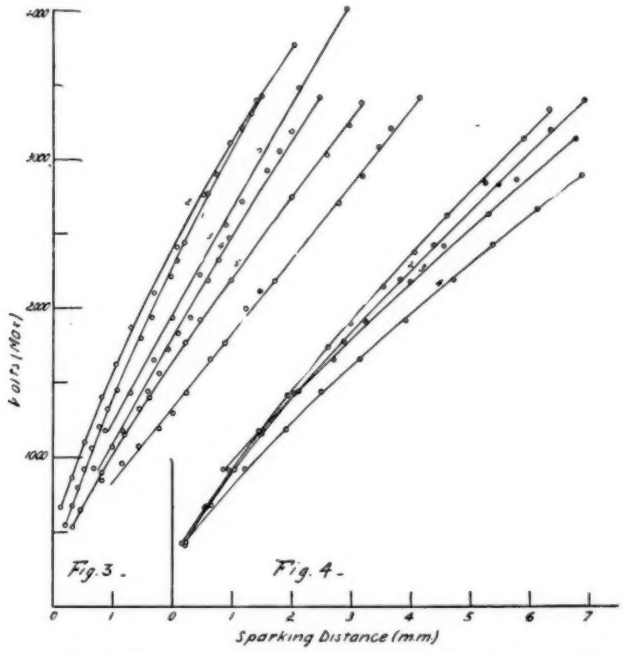

FIG. 3. Typical A.C. curves. 1. Al spheres (diam. 1 cm.) in air (press. 15 cm.). 2. Al spheres (diam. 1 cm.) in CO_2 (press. 15 cm.). 3. Al planes in air (press. 15 cm.). Ionizing agent: RaBr. 4. Al planes in air (press. 15 cm.). 5. Al spheres (diam. 1 cm.) in air (press. 10 cm.). 6. Al planes in air (press. 10 cm.).

FIG. 4. A.C. curves for hydrogen. 1. Al planes in H (press. 15 cm.). 2. Al spheres (diam. 1 cm.) in H (press. 15 cm.). 3. Cu spheres (diam. 2 cm.) in H (press. 15 cm.). 4. Al spheres (diam. 1 cm.) in H (press. 10 cm.).

pressure of 15 cm. Representative results are given in Tables I., II., and III., and in Figs. 3 to 9 inclusive.

A striking feature of all the curves is the increase in the potential gradient as the spark length decreases. Based on the work of Russell[1] and Dean,[2] Peek[3] gives the equation,

$$g_{max} = Ef_0/X,\ KV/cm,$$

in which g_{max} is the gradient across a grounded sphere-gap in air, E the

[1] Russell, Phil. Mag., 11, p. 237.

[2] Dean, PHYS. REV., 35, p. 468.

[3] Peek, Dielectric Phenomena, p. 30.

applied potential, X the shortest distance between the electrodes and f_0 a function of the spark distance and the radius of the spheres. For ungrounded spheres,

$$f = \frac{X/R + 1 + \sqrt{(X/R + 1)^2 + 8}}{4},$$

and Peek shows that this holds approximately for grounded gaps also. Table IV. shows the result of a few calculations made on this basis from experimental data.

TABLE IV.

Potential.	Distance.	Gradient.	Data.
3,295 A.C.	2.949 mm.	12,290 volts/cm.	Cu Sph. in CO_2
2,939 D.C.	3.000	10,950	" " "
2,665 D.C.	2.708	10,800	" " "
3,030 A.C.	2.708	12,280	" " "
2,752 A.C.	2.378	12,540	" " "
2,482 D.C.	2.378	11,310	" " "
1,221 D.C.	.754	16,630	" " "
1,389 A.C.	.754	18,880	" " "
3,310 A.C.	3.360	11,060	" " air
2,400 A.C.	2.110	12,200	" " "
1,914 A.C.	1.470	13,680	" " "
1,420 A.C.	.890	16,450	" " "

Peek[1] accounts for this increased gradient with decreasing spark length on the ground that the conductors are so close together that the ion accelerating or energy storage distance is interfered with, requiring an increased gradient to produce sufficient ionic density. Almy[2] has shown that with direct current, a discharge does not occur under 340 volts in air. Values of the same magnitude were obtained with both Cu and Al spheres in H with alternating potential in the present experiment. Russell[3] objects to the energy storage view and accounts for the potential gradient as being caused by this minimum sparking potential. It does not, however, seem from the form of the curves, that this explanation is sufficient, for, on Russell's theory the curves could then as well be straight lines. Hayden and Steinmetz[4] give another explanation, namely; that when the available energy is limited the striking distance becomes a function of the energy rather than of the potential, the expression given being,

$$W = .3d_0^{1.5},$$

[1] Peek, Am. Inst. E. E. Proc., 32 (2), p. 1774.

[2] J. E. Almy, Phil. Mag. (6), 16, p. 456.

[3] Russell, Electrician, 62, p. 894.

[4] Hayden and Steinmetz, Am. Inst. E. E. Proc., 29, p. 1151.

for 3.8 cm. spheres in air, where W is the energy of the discharge in joules and d_0 the distance in cm.

It will be noted that all of the graphs given here show (Figs. 5–8) that the alternating maximum potential is higher than the direct, and that the relative differences are the same for all gases investigated, and also for the different electrode materials. Whitehead[1] explains this difference as due to a cumulative energy effect in producing ionization while Hubbard[2] believes that the lag interval is the time required to convert a gas from an insulator into a conductor. Peek[3] states that there is a given minimum time in which the rupturing energy must be applied to the dielectric, and that a steep wave front requires a higher potential than a flat one. Fig. 10 shows the wave form at different

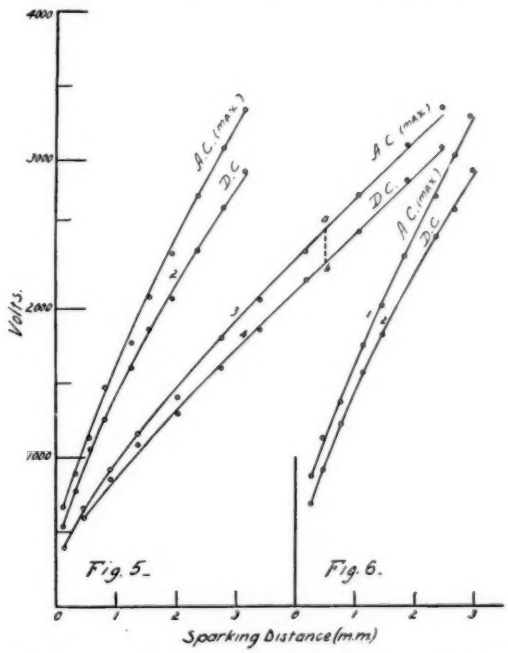

Fig. 5. A.C. and D.C. curves for H. 1. Cu spheres (diam. 2 cm.) (press. 30 cm.) A.C. 2. Cu spheres (diam. 2 cm.) (press. 30 cm.) D.C. 3. Cu spheres (diam. 2 cm.) (press. 15 cm.) A.C. 4. Cu spheres (diam. 2 cm.) (press. 15 cm.) D.C.

Fig. 6. A.C. and D.C. curves for CO_2. 1. Cu spheres (diam. 2 cm.) (press. 15 cm.) A.C. 2. Cu spheres (diam. 2 cm.) (press. 15 cm.) D.C.

amplitudes for the potentials used in this work. These curves were obtained by measuring the potential at points 5° apart on the circular scale attached to the Joubert disk for different values of the maximum

[1] Whitehead, Am. Inst. E. E. Proc., 29 (2), p. 1185.

[2] Hubbard, PHYS. REV., 32, p. 565.

[3] Peek, Dielectric Phenomena, p. 108.

potential, — 180° on the disk representing one cycle. The disk was mounted on a synchronous motor making 1800 R.P.M. This method of measuring the applied potentials has a distinct advantage over that used by some other observers, viz., multiplying the amplitude of the

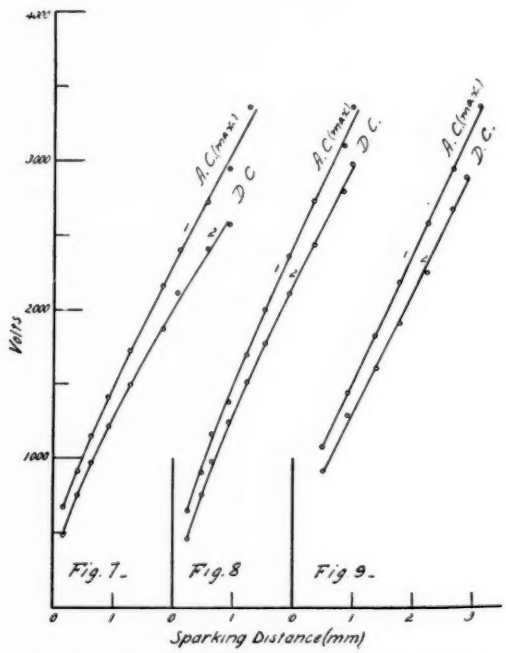

Fig. 7. In air. Cu spheres (diam. 2 cm.) (press. 15 cm.).
Fig. 8. In nitrogen. Cu spheres (diam. 2 cm.) (press. 15 cm.).
Fig. 9. In hydrogen. Al spheres (diam. 2 cm.) (press. 30 cm.).

primary potential by the ratio of transformation, in that it gives values directly and with a high degree of accuracy.

The curves show the presence of two principal secondary harmonics, and show that the wave form is practically constant for all potentials.

If, in any set of curves as shown in Figs. 5–8, the alternating and direct potentials corresponding to a given spark length be compared, and the time calculated from the wave form during which the alternating potential is above the value required to produce a spark of the same length with direct potential, the interval required to supply ionizing energy to the dielectric is obtained. For example the maximum potential shown by Curve 2, Fig. 10, is 2,550 volts. Referring to (*a*), (*b*), Fig. 5, we find that the D.C. spark potential corresponding to an A.C. spark potential of 2,550 volts is 2,300 volts. The time interval between the points (*c*), (*d*), Fig. 10, during which the A.C. potential is above 2,300 volts is

found to be 23.5°, which corresponds to .00218 sec. Table V. shows a series of such computations. It is evident that the variation from the

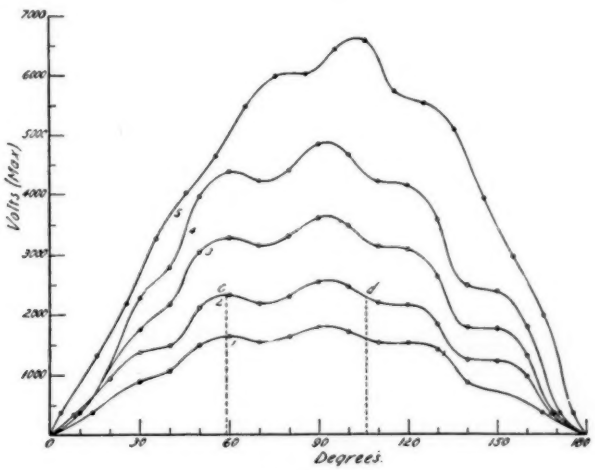

Fig. 10.

Wave forms. 1, 2, 3, 4. Open core transformer. 5. Potential transformer.

mean is small considering the difficulties connected with the method used for its determination.

TABLE V.

Data.	Pressure.	Potential.	Interval.
Cu Sph.	Air 15 cm.	2,520 A.C.$_{max.}$	2,220 D.C. .00245 sec.
" "	" "	1,760	1,520 .00263
" "	N "	2,550	2,280 .00231
" "	" "	1,760	1,560 .00245
" "	H 30	2,550	2,200 .00259
" "	" "	1,760	1,600 .00241
" "	" 15	2,550	2,335 .00208
" "	" "	1,760	1,590 .00231
" "	CO_2 "	2,550	2,280 .00231
" "	" "	1,760	1,570 .00241
Al Sph.	H "	2,550	2,300 .00199
" "	" "	1,750	1,630 .00221
" "	" 30	2,550	2,260 .00236
" "	" "	1,750	1,510 .00268
" "	N 15	1,750	1,530 .00264
" "	Air "	2,550	2,340 .00213
" "	" "	1,750	1,520 .00263
			Mean = .0024

If the time required for energy absorption is the correct explanation of the relation of alternating to direct spark potentials, then all maximum

potentials should lie above the direct, and mean effective values would be significant only when they include a reference to the wave form. Warburg[1] by suddenly applying direct potentials, was able to obtain a spark from an illuminated electrode in .0012 sec., or about one half the time interval of Table V. This is not necessarily contradictory evidence, however, for it is well known[2] that the sudden application of a potential has the effect of increasing the striking distance because of the large transient voltages produced. This fact was repeatedly observed in the present experiments, both with direct and alternating potentials. In fact modifications had to be made in the method of procedure in order to avoid these high, transient voltages. Kowalski and Rappel[3] give mean effective values which, if multiplied by the usual amplitude factor, 1.414, would lie considerably above corresponding direct potentials, but here again the real value of the comparison depends on the wave form.

If, however, the essential difference between alternating and direct spark potentials, as above stated, lies in the time required to transmit an amount of energy to the gas sufficient to produce sufficiently great ionization, the frequency of alternation should affect the maximum potential value. But Toepler[4] found no appreciable variation with frequency up to 10^6 cycles per second and irregular values above that number; and Whitehead and Gorton[5] observed only a 3 to 4 per cent. increase in corona voltages at 200 cycles as compared with 60 cycles. Yet in such cases and in all experiments with oscillatory potentials, the conditions approximate those of sudden impulse with high maximum gradient, while with a broad wave such as the one here used they are more nearly those of a gradually increasing electric force. Further investigations of this point are contemplated.

While this work was in progress E. Merkel[6] published an article covering to some extent the same ground. This investigator so regulated his power supply that he could produce discharges of large current density in the secondary circuit, one of those recorded being 1.45 Amp. $_{\text{eff.}}$ Because of the peculiar form of his curves (see Fig. 11) an effort was made to duplicate his results. Observations were made with different gases, at various pressures, and with both plane and spherical electrodes. The curves in Figs. 3 and 4 represent a few of the results obtained and are typical of them all. In these investigations the reactance and

[1] Warburg, Sitz. Akad. der Wissenschaften, XII., p. 223.
[2] Kemp and Stephens, Am. Inst. E. E. Jour., Jan., 1910.
[3] *Loc. cit.*
[4] *Loc. cit.*
[5] Whitehead and Gorton, Am. Inst. E. E. Proc., 33, p. 934.
[6] E. Merkel, Ann. der Physik, 43 (5), p. 725.

resistance in the spark circuit were such that the current for the highest potentials was about .007 ampere. For lower potentials the current following the advent of the spark became less and less until too small for accurate determination with the milliammeter used. In all of these cases the relation of potential to spark length was nearly linear, and in no case was there any suggestion of the anomaly as shown by Merkel. The steadying effect of illumination by ultra-violet was used in both cases.

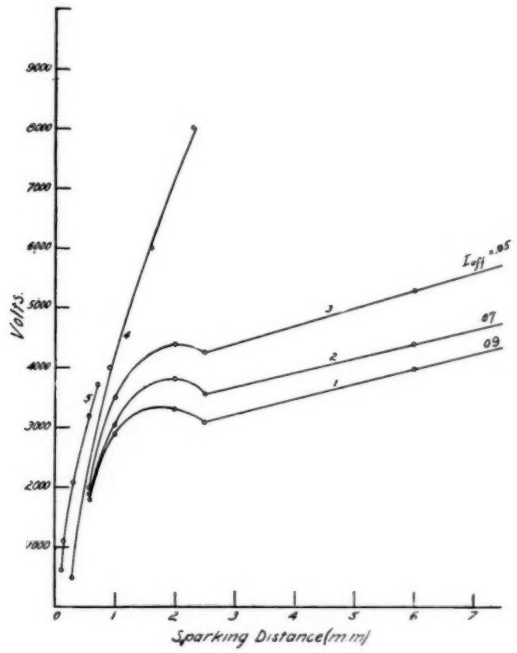

Fig. 11.

Air at normal pressure. 1. Cu electrodes—A.C. (effective), .09 amp. (eff.). Merkel. 2. Cu electrodes—A.C. (effective), .07 amp. (eff.). Merkel. 3. Cu electrodes—A.C. (effective), .05 amp. (eff.). Merkel. 4. Cu electrodes—D.C. Merkel. 5. Al electrodes—A.C. (max.), pressure, 73.9 cm. Jensen.

The only differences in our conditions of operation appear to be in the current density of discharge for various potentials—his being much larger than mine and constant throughout a set of potential-spark length determinations.

In order to obtain greater current densities than were possible with the open-core transformer used for the results so far given, two potential transformers of the closed-core type were used. These were so arranged that their secondaries could be connected either in series or in multiple, making possible secondary currents up to .1 ampere. The wave form of this combination is shown in graph number 5, Fig. 10. It having

been found impracticable to duplicate Merkel's curves—(1, 2, 3, Fig. 11) with respect to constant current densities over their entire range, the problem was attacked from a different angle. Reference to these curves shows that for a 1.5 mm. spark in air at atmospheric pressure the potential should vary by 15 to 20 per cent. if the current density of the discharge be doubled. In Table VI., is shown a series of results obtained

TABLE VI.

Al Spheres. 1 cm. Radius in Air. Pressure 73.5 cm.

Potential.	Spark Length.	Current.
6,810 A. $C._{max.}$	1.390 mm.	.0275 I_{eff}
6,810	1.410	.1030
6,810	1.340	.0424
6,810	1.365	.0280
5,380	1.067	.0280
5,380	1.030	.0080
5,230	1.015	.0370
5,230	1.080	.0120
4,190	.940	.0125
4,190	.990	.0070

with air at atmospheric pressure, the currents and spark lengths being of the order of those involved in the portion of Merkel's curves under discussion. Observations were readily reproducible, the values given being in most cases averages of five sets of readings. In each pair of observations the conditions as to temperature, polish of electrodes, etc., were as nearly identical as possible.

One of the characteristics of Merkel's observed anomaly is that with increasing current density of discharge, the necessary potential decreases. Referring to Table VI., it will be noted that for a given potential the spark length remained practically constant even for a range of current densities from one to four times the lower value, the gas, electrodes, and pressure remaining unchanged. Since Merkel's observed variation was through only half this range, the explanation for the hump in his curves must be sought in another direction.

Merkel states that Guye and Monasch,[1] who had performed experiments in this field, explained this " critical zone " comprising the hump as due to resonance phenomena, a condition which he sought to obviate by the method he used to measure potentials. But a careful study of

[1] Guye and Monasch, L'Eclairage Elec., 34, p. 305; 35, p. 18.

his apparatus does not seem to substantiate this contention, for the capacity of the spark gap he used would be appreciable and would vary with the spark length, and the inductance introduced into the oscillatory circuit by his electrodynamometer must also be accounted for. Whitehead and Gorton[1] observed a resonance condition which persisted over a wide range, and Peek[2] called attention to the necessity of using a series resistance in the spark circuit to prevent irregular effects due to high, transient voltages. The writer found evidence of such disturbances in a few instances only, when working at the lower pressures and greater spark lengths; also with the higher potentials when using the larger transformers. This would seem to indicate that the capacity here used was smaller than Merkel's. The disturbance caused by the static spark mentioned by Peek was also noticed, particularly when the liquid resistance in the circuit was much reduced in order to make observations with large current density. In the intensely lighted gap it was often impossible to see these minute sparks, but by the use of the telephones they could be easily heard, a considerable number of them sometimes passing before the true disruptive spark, capable of affecting the ammeter, was heard. That these were not point discharges due to an electrode surface roughened by repeated heavy sparks is shown by the fact that the phenomenon also occurred before the first disruptive discharge while the surfaces were still highly polished. In fact, the electrodes were not permitted to become appreciably disfigured, and an examination of the planes showed that the discharge took place fully as much in the center of the exposed circle as at the edges of the glass cover.

Oscillatory conditions such as those above mentioned were probably present in the apparatus used by Merkel, a conclusion which is fully justified by a study of his oscillograph curves. Inasmuch as the anomalous condition he observed is contrary to generally accepted principles, this explanation seems at least a plausible one.

Summary.

1. In the present investigation alternating and direct spark potentials have been measured under exactly the same conditions—the gas, electrodes, and pressure being identical throughout.

2. It has been shown that the effect of illumination by ultra-violet light of a gap supplied with alternating potential is essentially the same as that observed by Warburg for direct potential; and, in addition, that the presence of an abundance of ions lessens the irregular influence of transient voltages due to static and resonance phenomena.

[1] Whitehead and Gorton, Am. Inst. E. E. Proc., 33, p. 934.

[2] Peek, Am. Inst. E. E. Proc., 33, p. 911.

3. The time interval during which the alternating potential must be above that necessary to produce a spark by direct potential was found to be practically the same for all conditions as regards gases, electrodes, and pressures, the alternating maximum potential always lying above the corresponding direct one.

4. An attempt to reproduce evidence that, with an increase of current density of discharge, there is a corresponding decrease of spark potential, gave negative results, all indications tending to show that such an effect must be due to resonance in the discharge circuit.

In conclusion, the writer wishes to express his thanks to Professor Skinner for suggesting this investigation and for his continued interest in its progress; and to Professor Almy and Dr. Tate for valuable suggestions in overcoming experimental difficulties.

BRACE LABORATORY OF PHYSICS,
LINCOLN, NEB.,
April 1, 1916.

THE NATURE OF THE COLLISIONS OF ELECTRONS WITH GAS MOLECULES.

BY K. T. COMPTON AND J. M. BENADE.

INTRODUCTION.

A STUDY of the collisions between electrons and gas molecules should yield information regarding molecular and atomic structure, since the phenomena accompanying such collisions must be explicable in terms of the distribution of electric charges in the molecule and the resulting electromagnetic field. Only a few characteristics of such collisions are well established, but these are very suggestive.

When electrons or α particles are projected with high speeds into matter their paths occasionally bend through large angles, indicating that they have passed through regions of intense force. Experiments by Rutherford,[1] Geiger,[2] C. T. R. Wilson[3] and others, interpreted by Rutherford's theory of single scattering,[4] have indicated that each atom contains a charged nucleus of excessively small dimensions but of a mass practically equal to that of the atom. Such rapidly moving electrons or α particles are not useful in investigating the conditions of the electrons surrounding the nucleus, since their great momentum carries them past such mobile obstacles without appreciable deflection.

Slow moving electrons may be deflected by the electrons surrounding the nucleus and may give up to these electrons a part of their energy. The scattering, energy loss and ionization by slow moving electrons is therefore more complicated than in the case of high speed particles, but from their study we may hope to gain additional knowledge of atomic structure.

A series of investigations by Franck and Hertz[5] has proved that collisions of slow moving electrons with molecules of the inert gases helium, neon, etc., and of mercury vapor are nearly or entirely elastic, while in the common gases the electrons lose a large portion of their energy at each collision. Hydrogen molecules appear to be the most nearly

[1] Phil. Mag., 11, p. 166, 1906.
[2] Proc. Roy. Soc., A, 81, p. 174, 1908; 83, p. 492, 1910.
[3] Proc. Roy. Soc., A, 87, p. 227, 1912.
[4] Phil. Mag., 21, p. 669, 1911.
[5] Verh. d. D. Phys. Ges., 15, pp. 34, 373, 613, 1913; 16, pp. 12, 457, 1914.

elastic and the heavy molecules the most perfectly inelastic of the common gases. These experiments and those by Lenard[1] and Pawlow[2] show that it is impossible for an electron to ionize a gas molecule unless its velocity at impact exceeds that acquired by motion through the minimum ionizing potential V_0 which is characteristic of the gas.

When we consider the number of collisions made by an electron in a given path through a gas we are confronted by the problem of defining the meaning of a collision. In experiments on scattering we should define a collision as a deflection through an angle greater than some arbitrary critical angle, while in experiments on ionization we might define it as a loss of energy exceeding some arbitrary critical amount. Following the ordinary gas theory the theories of ionization by collision have supposed the molecules to be impenetrable regions, for convenience assumed spherical. It is evident that equations based on such an hypothesis can only represent an average state of affairs, and if the radii of these hypothetical spheres are calculated from data of ionized gases they will be such as to make the total energy lost by an electron in a centimeter path equal to the actual loss as it moves one centimeter among the force centers.

If r is the radius of one of these hypothetical spheres (molecules) and if there are n molecules per unit volume, $\nu = \pi r^2 n$ gives the average number of collisions made by an electron while moving one centimeter through the gas. If we take r to be the value of the molecular radius calculated from the coefficient of viscosity[3] we obtain the values of ν given in the first column of Table I. If we calculate ν from the mean free path of a gas molecule L, since Maxwell has shown that the mean free path of a very small particle moving with relatively high speed among gas molecules is $4\sqrt{2}L$, we obtain the values given in the second column. The values of ν resulting from the theories of ionization proposed by Townsend, Bergen Davis and one of the writers are given in the last three columns.[4] The values are given in terms of N, which is the value of ν when the pressure is 1 mm. In general $\nu = pN$.

Our original purpose in this investigation was to make direct measurements of N. If we assume, as is done in the theories of ionization by inelastic impact, that the electrons start approximately from rest after each collision and move freely under the influence of the electric field until the next impact, it should be very easy to determine N by the aid of the apparatus described below.

[1] Ann. d. Phys., 8, p. 190, 1902.

[2] Proc. Roy. Soc., A, 90, p. 398, 1914.

[3] Kaye and Laby's Tables, p. 33.

[4] These values are taken from an earlier paper, PHYS. REV., 7, p. 507, 1916, and depend somewhat on the choice of data used in determining them.

TABLE I.

Gas.	$\pi^2 rn$	$1/4\sqrt{2}\,L$	Townsend.	Davis.	Compton.
N_2	34.5	24.7	12.4	17.6	22.5
CO_2	49.5	36.9	20.0	23.2	30.0
H_2O	47.0	32.1	12.9	15.9	21.7
HCl	46.0	33.8	22.2	26.8	33.3
He	13.5	8.3	—	—	10.0

APPARATUS.

The general plan of the experiment was to project electrons with a definite velocity through a hole in a plate so that they moved toward a second parallel plate against a retarding field which was insufficient to prevent their reaching the plate unless they collided on the way. From the relative numbers reaching the second plate at various distances from the first plate the average number of collisions per centimeter path by each electron could be easily calculated.

The source of electrons was a fine platinum strip *F*, Fig. 1, which was coated at the middle with a thin layer of lime from sealing wax. These electrons were drawn upward by an accelerating field of *U* volts toward the plate *B*. This plate was provided with two holes of a millimeter diameter, separated by a vertical distance of one centimeter, the lower one being about three millimeters above the middle of the filament. The purpose of this arrangement was to insure an approximately parallel beam of electrons projected into the space above the plate and to protect the upper region from any field due to the potential below. Above the plate *B* the electrons were subjected to a retarding field *V*, which was less than *U*, so that they would reach the receiving plate *A* unless they collided with a gas molecule in the intervening space. If they collided, and consequently lost their upward momentum, the field *V* pulled them back to the plate *B*. The plate *A* could be raised or lowered by a winch device *W* through a distance of about 15 cm. According to the ordinary kinetic theory, the number of electrons *n* reaching the

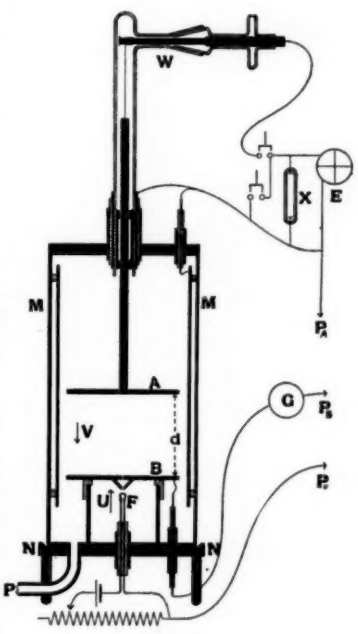

Fig. 1.

plate A in a given time should bear to the number n_0 passing through the hole the relation

$$n = n_0 e^{-pNd}, \tag{1}$$

where d is the distance between the plates and $pN = \nu$ is the average number of collisions by each electron in a centimeter path. From these measurements N, the value of ν for one millimeter pressure, could be calculated. In practise it was easier to use the equivalent relation

$$\frac{n_1}{n_2} = e^{-pN(d_1-d_2)}, \tag{2}$$

where the subscripts refer to measurements at two different distances.

Measurements of n were made by a Dolazalek electrometer E, which was used at various values of the sensitiveness between 1,000 and 10,000 divisions per volt. In the earlier experiments the rate of deflection method was employed, while later the quadrants were shunted by a xylol resistance X, as shown in the figure. It was found that a fairly uniform electronic current was obtained from the filament after it had been heated for a couple hours following the exhaustion of the apparatus. In order to guard against possible errors from variations of the electronic current, a Leeds and Northrup high sensitivity galvanometer G, with a sensitiveness of 6.7 $(10)^{-10}$ ampere per scale division, was connected to measure the current to the plate B. For large variations in the electronic emission the ratio of the electrometer to the galvanometer readings was not constant, but it was sufficiently constant for our purpose in view of the small variations during any one set of measurements. As a further precaution against these variations and those due to the slow vacuum leak, which we were unable to entirely eliminate, the measurements of n_1 and n_2 were made alternately in rapid succession and the average value of the ratio was used to determine N.

The potentials of all parts of the system were fixed by three separate potential dividing sets P_A, P_B, P_F. The potential of the filament was taken to be that of its middle point. The accelerating potential drop U was always less than the minimum ionizing potential of the gas.

The apparatus was located inside a large brass pot MN, whose removable cover MM was sealed to the base NN with a rosin-beeswax mixture. Guard ring devices protected all points where electric leaks would be disastrous, and effects from contact difference of potential were avoided by making all parts of brass. The entire system was placed inside an earthed metal cage. The apparatus was exhausted through P by a Gaede mercury pump and the pressure, which was usually between 0.01 and 0.05 mm., was measured by a McLeod gauge.

Phosphorous pentoxide and gold coated glass beads were introduced in the connecting tubes to eliminate moisture and mercury vapor. All the experiments here recorded were performed with nitrogen, prepared from sodium nitrite and ammonium chloride.

In the early experiments in which the rate of deflection method was employed, account was taken of the variation in the capacity of the electrometer system as the position of the plate A was changed.

EARLY EXPERIMENTS AND CONCLUSIONS.

A large number of determinations of N were made under different conditions of pressure and accelerating and retarding field, yielding apparently equally reliable values of N ranging between 39.5 and 10.3. The mean value of all the determinations was 18.7. These are of the order of magnitude of the values in Table I., but the average value is of no particular significance, since it depends on the experimental conditions under which the majority of the observations were taken. The large variation in these values cannot be attributed to experimental error, since individual measurements could be duplicated very exactly.

In order to find the factors on which N depends, series of readings were taken keeping all but one of the experimental conditions constant. In this manner it was found that (1) the value of N calculated by equation (2) increases and approaches a constant value as the distance between the plates A and B is increased; (2) this constant value is approached more rapidly at high than at low pressure; (3) this constant limiting value of N is a function of the difference between the accelerating and retarding potentials $(U - V)$, N increasing as $(U - V)$ decreases.

Apparently N is also a function of the velocity of the electrons, since we could obtain a regular and consistent variation of N with $(U - V)$ only provided U was maintained constant. We have not discovered the law of this variation and we therefore kept U constant in experiments relating to the variation of N with $(U - V)$.

Examples of these variations are given in Fig. 2. Here the natural logarithms of the electrometer deflections, reduced to a convenient scale, are plotted as the values of $\log n_1$, $\log n_2$, etc.; against the corresponding distances d_1, d_2, etc. The value of N corresponding to any point on a curve is found by dividing the slope of the curve at that point by the pressure.

Probably the only way of reconciling these variations with the invariable value of N required by the concept of a collision which we have used thus far was to explain them as being due to variations of the

electron reflection from the plate A, whereby the proportion of incident electrons actually caught by the plate might depend on the velocity of impact or the electric intensity in the region AB. We tested this possibility both by experiments in which one or the other of these factors was maintained constant and by computations in which factors were introduced to allow for reflection, and found by both methods that our results are not in error from this cause. It should also be mentioned that neither the magnetic field of the earth nor of the heating current in

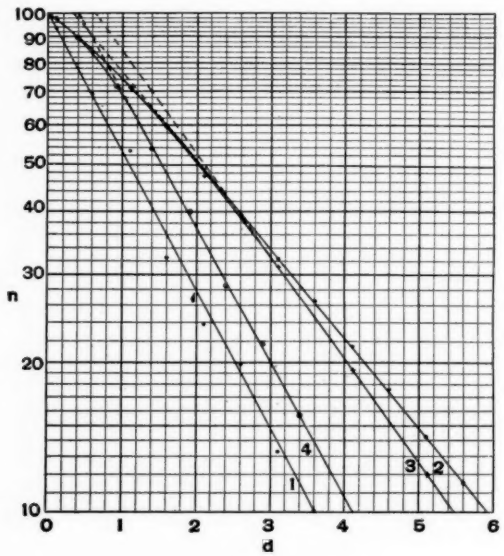

Fig. 2.

Effect of the distance d between the plates on the number of electrons n reaching the upper plate. Curve (1); $U = 6.32$ volts, $V = 6.0$ volts, $p = 0.027$ mm. Curve (2); $U = 8.12$ volts, $V = 6.0$ volts, $p = 0.025$ mm. Curve (3); $U = 9.32$ volts, $V = 8.5$ volts, $p = 0.019$ mm. Curve (4); $U = 8.13$ volts, $V = 6.0$ volts, $p = 0.042$ mm.

the filament is capable of accounting for the variations since both are much too small to produce an appreciable effect in the region above B, and even if such an effect did exist it would cause N to vary with d in a manner opposite to that observed.

In view of these results it seems necessary to reject the simple concept of a collision on which we have based our work up to this point, since this concept requires that N be a constant quantity. If the assumptions regarding collisions that have been made in the theories of ionization by collision were correct, curves such as those in Fig. 2 should be straight lines with equal slopes under all conditions of velocity, distance. The most definite deduction from these experiments is that

we cannot consider a nitrogen molecule to be a definite region which stops those electrons which strike it and allows those which do not strike it to pass by freely. It appears necessary to revise equations (1) and (2) to include the possibility that an electron may collide, in the sense to which N refers, and still reach the plate A. This revision may be made as follows.

FORMULA FOR N EXTENDED TO INCLUDE THE POSSIBILITY OF AN ELECTRON REACHING THE PLATE A IN SPITE OF A COLLISION.

Suppose we now define a collision as occurring whenever an electron approaches so close to the center of a molecule that it *may* be prevented from reaching the plate A. In other words, we imagine the molecule to lie inside a certain spherical region of such a radius that no electrons passing outside this sphere lose enough upward momentum to prevent their reaching the plate A, while any electron passing into this sphere may or may not be stopped, depending possibly on the orientation of the molecule or of some of its parts. The average number of collisions per centimeter path by an electron is therefore $\nu = \pi r^2 n$, where r is the greatest distance from the center of a molecule at which a passing electron can be so deflected or stopped that it fails to reach the plate A. We shall see later that r, and therefore ν, is a function of $(U - V)$, *i. e.*, of the energy which must be lost by an electron at a collision in order that it may be prevented from reaching A. For the present we shall consider U and V to be kept constant and see how the number of electrons which reach the plate A varies with the position of the plate.

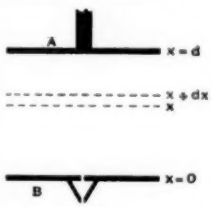

Fig. 3.

Let s represent the probability that an electron which has collided (in the sense defined above) may reach the plate A. s has heretofore been supposed to equal zero. If n electrons per second enter a layer dx between A and B (Fig. 3), there are $n\nu dx = npNdx$ collisions in dx per second. Thus the number of electrons which collide in dx per second and are thereby prevented from reaching A is

$$dn = - npN(1 - s)dx.$$

The value of s may depend on two factors, loss of energy and deflection at a collision. Let M be the probability that, in spite of the collision, the electron will still retain sufficient upward momentum to reach A. The probability of getting to A without another collision is $e^{-pNL(d-x)}$, where L is the ratio of the average path of those reaching A after a deflection to the direct path $(d - x)$. If we assume, as a first approxima-

tion, that no electrons can reach A in spite of more than one collision, we have

$$s = Me^{-pNL(d-x)};$$

whence

$$dn = -npN(1 - Me^{-pNL(d-x)})dx,$$

or

$$n = n_0 e^{-pNd+(M/L)(1-e^{-pNLd})}. \tag{3}$$

Similarly, if we assume that, as a closer approximation, no electrons can reach A after more than two collisions, we find

$$n = n_0 e^{-pNd+(M/L)(1-e^{-pNLd}+(M'L/L')(1-e^{-pNL'd}))}. \tag{4}$$

In the cases to which equations (3) and (4) were applied the calculated values of n agreed within one or two per cent. for all values of pd, so that equation (3) may be considered a sufficiently close approximation for testing the experimental results. The experimental tests indicate that the extension of the idea of a collision embodied in these equations is a step in the right direction.

VERIFICATION OF EQUATION (3). VARIATION OF n WITH pd AND OF N WITH $(U - V)$.

It is immediately evident that equations (3) or (4) are in agreement with the experimental data and may be made to fit them by properly choosing the constants M, L and N. When the product pd is comparatively large the variation of n with d predicted by these equations becomes practically identical with that predicted by equation (1). This accounts for the observation that the experimental curves approach a constant slope as the distance increases, approaching it more rapidly at the higher pressures. In Fig. 4 are shown curves representing equation (3) plotted with various arbitrary values of M, L and N. It is found, for instance, that curve (2) of Fig. 2 coincides with equation (3) if we take $M = 0.4$, $L = 1.9$, $N = 17.0$.

The theoretical significance of this agreement is that some electrons really do reach the plate A in spite of a collision in the retarding field: its practical importance lies in the fact that the true value of N may be calculated by simply applying equation (2) to the experimental curve in the region where it is approximately a straight line.

Having found this simple and consistent method of determining N, we next sought the law of variation of N with $(U - V)$. To do this we made several sets of measurements at various pressures and accelerating fields U. In each series the values of N were determined for various values of the retarding potential V by applying equation (2) to measure-

ments of n taken at distances large enough to insure their falling on the straight portion of the curve.

It was found that the variation of N with $(U - V)$ could be best represented by an expression of the form $(U - V) = 1/N^5$. This is shown by the experimental data represented in Fig. 5, in which corresponding

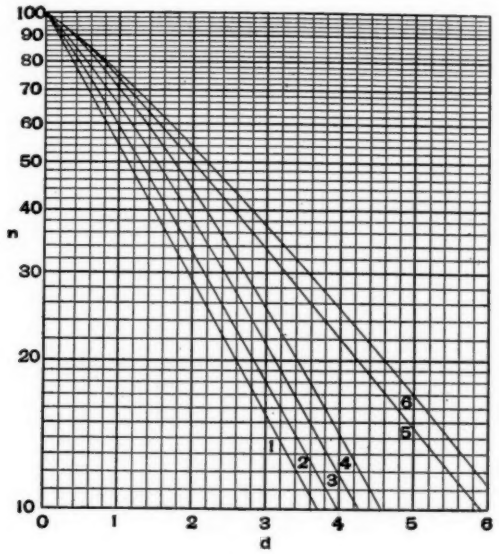

Fig. 4.

Graphs of equation (3). Curve (1); $M = 0$, $L = 1$, $N = 25$. Curve (2); $M = 0.2$, $L = 1$, $N = 25$. Curve (3); $M = 0.4$, $L = 1$, $N = 25$. Curve (4); $M = 0.6$, $L = 1$, $N = 25$. Curve (5); $M = 0.4$, $L = 1$, $N = 17$. Curve (6); $M = 0.4$, $L = 2$, $N = 17$.

values of N and $(U - V)$ are plotted on logarithmic paper. The upper curves, (1) and (2), represent the two most consistent sets of measurements, while the lower curve (3) represents the average of all our determinations, each type of dot representing a separate series of measurements. The slope of these curves, denoted by s, gives the exponent in the power law $(U - V) = 1/N^s$, which is found to be $s = 5$.

The accuracy which we were able to attain in these tests is not very great; yet we feel reasonably safe in concluding that this fifth power variation must be at least very nearly correct. In the following discussion it will be seen that the nearest other power variation having theoretical significance is the case where $s = 2$. Curve (4) represents such a variation, which is clearly not in accord with the experimental data.

The fact that the different series of measurements do not coincide, but that each curve falls above or below others, we attribute to the

fact that they were taken with different pressures, accelerating potentials and heating currents. The variations may be due to a real variation of N with U or to differences in the velocity distributions of the electrons in the various cases. In any given series of measurements, however, variations in these factors were eliminated as far as possible. For this reason we believe that the experiments determine the slope s more accurately than they fix definite corresponding values of N and $(U - V)$.

These experiments therefore indicate that N varies inversely as the

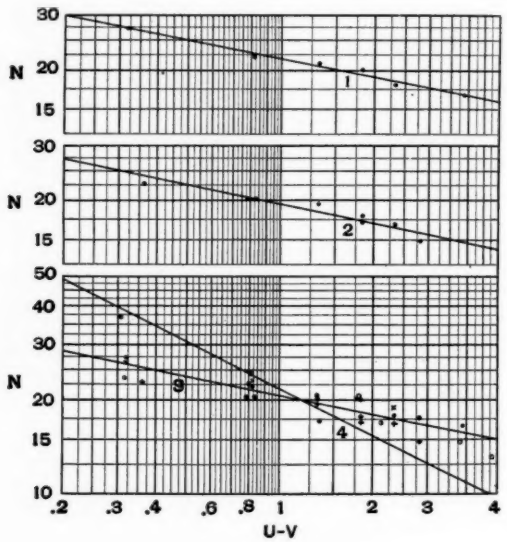

Fig. 5.

Variation of N with $U - V$, the slope s giving the exponent of the power law. Curve (1); $U = 8.32$ volts, $p = 0.025$ mm., $s = 5.1$. Curve (2); $U = 9.33$ volts, $p = 0.030$ mm., $s = 4.9$. Curve (3), $s = 5$. Curve (4), $s = 2$.

fifth root of $(U - V)$. Since $pN = \pi r^2 n$, this means that the electron must come within a distance r of the center of the molecule in order that it may lose energy proportional to $(U - V)$, and that as $(U - V)$ increases, r decreases according to the relation

$$(U - V) \propto \frac{1}{r^{10}}. \tag{5}$$

Let us now consider what characteristics of a molecule may give rise to such a relation.

THEORY OF COLLISIONS.

(*a*) *Forces Acting and Work Done.*—There are four distinct ways in which a passing electron may do work on a molecule and thence lose

part of its kinetic energy, even though the molecule is electrically neutral.

If the electron is moving with high speed it may approach so near an electron of the molecule that the mutual force of repulsion far exceeds the forces from the remaining parts of the molecule; or it may even penetrate an atom and pass close by the nucleus. In such cases we should expect the force to vary inversely as the square of the distance between the two charges, as has been shown experimentally by the scattering of high speed electrons or α particles. As a result of this force, work is done by the electron as it approaches and on the electron as it recedes from the repelling center and its original kinetic energy is not lost unless it leaves the molecule permanently altered either by ionizing it or by causing its parts to vibrate and emit radiations. Thus it is difficult to calculate the loss of energy resulting from forces of this type at collisions. In the case of slow moving electrons, such as we have been considering, it is practically certain that the repulsion due to the outer rings of electrons is sufficient to prevent the impinging electron from approaching near enough to render the work done by forces of this type as important as those which we shall now consider.

The neutral molecule may be an electric doublet, whose moment we may call μ. As an electron approaches the molecule along its axis from a great distance to the distance r, the work done is

$$-\int_{\infty}^{r} \frac{2\mu e}{r^3}\,dr = \frac{\mu e}{r^2}.$$

If the approach is not along the axis of the doublet the work is still inversely proportional to the square of the distance, though a numerical factor is introduced. Suppose that the fraction f_1 of this work represents energy lost by the electron before it leaves the neighborhood of the molecule. This loss might be due to its setting the molecule in rotation or to setting its parts in motion relative to each other. Thus we can let $f_1 A/r^2$ represent the energy lost by the electron due to the permanent electric field of the molecule.

In any case we believe that the electric field due to the passing electron displaces the electrons of the molecule, thus polarizing it and producing a doublet whose moment depends on the nature of the molecule and its distance from the electron. It is possible to calculate the magnitude of the force exerted on the electron by this induced doublet by considering the specific inductive capacity of the gas. The electric moment per unit volume of the medium, when placed in an electric field of intensity E is equal to the polarization

$$P = \frac{K-1}{4\pi} E.$$

If n is the number of molecules per unit volume, the electric moment of each molecular doublet is

$$\mu = \frac{P}{n} = \frac{K-1}{4\pi n} E.$$

In the present case the field E is that due to the electron at a distance r and is e/r^2. Thus the molecular moment is

$$\mu = \frac{K-1}{4\pi n} \frac{e}{r^2}.$$

The force of attraction between the electron and the molecule is

$$F = \frac{2\mu e}{r^3} = \frac{K-1}{2\pi n} \frac{e^2}{r^5}, \tag{6}$$

which is an inverse fifth power law of attraction.[1]

The work done on the electron by this force as the electron approaches to a distance r is given by

$$-\int_{\infty}^{r} \frac{K-1}{2\pi n} \frac{e^2}{r^5} dr = \frac{K-1}{8\pi n} \frac{e^2}{r^4} = \frac{B}{r^4}.$$

If we suppose that a fraction f_2 of this energy is permanently lost by the electron, we have $f_2 B/r^4$ for the loss of energy arising from the formation of the induced molecular doublets.

Finally, if the molecule consists of more than one atom, the induced electric moment of the nearest atom causes it to exert the greatest force on the electron, with the result that this atom is displaced toward the electron relatively to the rest of the molecule. For a small displacement this displacement must be proportional to the force, or $F = cs$. Since in this case the force on the molecule consists almost entirely of the force on the nearest atom, we have approximately, from equation (6),

$$F = \frac{K-1}{2\pi n} \frac{e^2}{r^5} = cs;$$

$$dF = c\, ds = -\frac{5(K-1)}{2\pi n} \frac{e^2}{r^6} dr.$$

The total work done in thus displacing the atom with respect to the molecule is

$$\int_{\infty}^{r} F ds = -\int_{\infty}^{r} \frac{5(K-1)^2}{4\pi^2 n^2 c} \frac{e^4}{r^{11}} dr = \frac{(K-1)^2}{8\pi^2 n^2 c} \frac{e^4}{r^{10}}. \tag{7}$$

If we let the fraction f_3 of this work represent energy permanently lost

[1] Langevin (Ann. de Chim. et de Phys., 28, p. 316, 1903) showed that forces of this nature would account for the accumulation of neutral molecules around electrons, resulting in the formation of ionic clusters and explaining the abnormally small rates of diffusion of the ions at high pressures and low temperatures.

by the electron, we may write f_3C/r^{10} for the energy lost by the electron due to this cause.

The total energy lost by the electron in approaching within a distance r of the molecule and again receding must therefore be given by

$$\Delta W = \frac{f_1 A}{r^2} + \frac{f_2 B}{r^4} + \frac{f_3 C}{r^{10}}, \tag{8}$$

where we have assumed that the electron does not possess kinetic energy sufficient to ionize the molecule or to enable it to penetrate an atom or approach very near one of its constituent parts.

Our experiments indicate that in nitrogen the third term in the right member represents the predominating factor in the loss of energy at a collision. This would probably be anticipated, since the second term involves a displacement of electrons whose natural period of vibration is much shorter than the time of approach and recession of the colliding electron, while the third term involves the displacement of an atom whose natural period, as we shall see, is comparable with the time of action of the colliding electron. There can therefore be no considerable lag, or hysteresis effect, in the second term, while such an effect may account for the energy loss represented by the third term. The work done in this displacement is likely of the same nature as that which is accounted for by frictional effects in the theory of absorption and dispersion. Thus f_2 is probably very small and f_3 may be comparable to unity. The first term involves no internal change in the molecule, but only a possible change in its motion.

(*b*) *Explanation of Elastic and Inelastic Collisions.*—It is significant that the third term in equation (8), which represents the predominating cause of energy loss at a collision in nitrogen, does not exist in the case of monatomic gases. We should therefore expect collisions in monatomic gases to be approximately elastic, *i. e.*, that very little energy would be lost by an electron at a collision. This has been found to be true.[1] Furthermore, in gases with light molecules, the third term would have a smaller value than in the case of heavy complex molecules, owing to the smaller relative displacement of the constituent atoms when polarized by the neighboring electron. In other words, the approximation which we made in deriving the third term of equation (8) is less justified in

[1] When Franck and Hertz discovered the property of elastic collision in helium, neon and argon, they considered it a characteristic of the inert gases. The later discovery that collisions in hydrogen were partially elastic led them to conclude that this elasticity is roughly proportional to the minimum ionizing potentials of the gases. The fact that mercury vapor, though easily ionized, is characterized by almost perfect elasticity of collisions, indicates that elasticity of collisions is a property characteristic of monatomic molecules, and to a less extent, of light molecules.

the case of a light atom than in the case of a heavy complex atom. This has also been verified by Franck and Hertz,[1] who estimated the average energy loss at a collision, for electrons with velocities less than the minimum ionizing velocity, to correspond to a motion through 0.3 volt in helium and 1.6 volts in hydrogen, and found that practically the entire energy is lost at collisions in oxygen and nitrogen.

NATURAL FREQUENCY OF VIBRATION OF AN ATOM.

These views regarding the nature of energy loss at a collision are supported by a consideration of the forces which hold atoms to their positions of equilibrium in molecules, as indicated by the natural frequencies of vibration of the atoms when displaced.

We have seen that the restoring force on an atom when displaced a distance s from its position of equilibrium is given by $F = -cs$. An approximate value of the constant c may be estimated from our experimental data as follows. If all the work done at a collision is represented by the third term of equation (8) we have, from equations (5), (7) and (8),

$$e(U - V) = \frac{f_3}{c} \frac{(K-1)^2}{8\pi^2 n^2} \frac{e^4}{r^{10}}.$$

Substituting N for r from $\pi r^2 n_1 = N$, where n_1 is the number of molecules per cubic centimeter at the pressure, one millimeter, for which N was calculated from our experiments, we have

$$\frac{c}{f_3} = \frac{K-1)^2 e^3 \pi^3 n_1^5}{8n^2(U - V)N^5}.$$

Under standard conditions $n = 2.705(10)^{19}$ and $K = 1.00058$ for nitrogen. At one millimeter pressure $n_1 = n/760 = 3.56(10)^{16}$, under which conditions we found $N = 20$ when $(U - V) = 1.16$ volts. Substituting these values and taking $e = 4.7(10)^{-10}$ e.s.u.; we find that

$$\frac{c}{f_3} = 0.852(10)^6.$$

Since the value of f_3 is probably near and cannot exceed unity, this number is the maximum possible value of the constant c which enters into the relation

$$M \frac{d^2 s}{dt^2} = -cs.$$

The natural period of the displaced atom is therefore $T = 2\pi\sqrt{M/c}$, which corresponds to radiation of wave-length $\lambda = 3(10)^{10}T$. Substi-

[1] Verh. d. D. Phys. Ges., 15, p. 373, 1913.

uting the mass of the nitrogen atom $M = 23.2(10)^{-24}$ gm. and the value of c just calculated we find

$$\lambda = 0.00098 \text{ cm.}$$

for the wave-length of the radiation characteristic of the nitrogen atom. From the assumptions underlying its derivation, we should expect this to be the minimum possible value of the characteristic wave-length.

As a matter of fact this is of the order of magnitude of the wave-lengths which we think of as due to the vibrating atoms. E. R. Drew[1] found a very strong absorption band in nitrogen in the neighborhood of $\lambda = 0.00047$ cm. Niels Bjerrum[2] applied the quantum theory to the specific heats of gases in the same manner as Einstein and Nernst-Lindemann have applied it to solids. Following Einstein's method he found $\lambda = 0.00036$ cm. for the wave-length of the radiation from a nitrogen atom vibrating in the direction of its chemical bond.

While the agreement is good only as regards order of magnitude, still it is quite satisfactory when one considers the possible experimental errors in our determinations and when one remembers the round-about method in which the calculation had to be made. Taking everything into account we feel that the evidence supports the theory of collisions which we have proposed.

BEARING OF THESE RESULTS ON THEORIES OF IONIZATION BY COLLISION.

The results of this investigation have no bearing on the theory of ionization of gases by elastic impact, but they render untenable the simple assumptions regarding collisions on which the theory of ionization by inelastic impact was based. Yet the accuracy of the equation for inelastic impact is such that we must accept it as being very nearly correct. The theory of ionization may be reconciled with the present results by supposing that the average amount of energy lost by an electron in moving one centimeter among the force centers is the same as if it lost its entire energy at each impact with imaginary spheres of definite radii. The concept of a collision in the ionization theories is, therefore, an artificial one, which is made to yield accurate results by arbitrarily ascribing to the molecules such dimensions as will make the calculated loss of energy equal the actual loss of energy as an electron moves through the gas. We shall not attempt in this paper to modify the equations for ionization by collision so as to incorporate the theory of collisions developed here.

[1] PHYS. REV., 21, p. 122, 1905.

[2] Zeits. f. Elektrochemie, 17, p. 731, 1911.

SUMMARY.

In order to interpret the experiments recorded in this paper it has been found necessary to suppose that

1. An electron is not always stopped whenever it strikes the spherical region which we sometimes call the molecule; whence we conclude that the molecule, or its field of force, does not possess radial symmetry.

2. The energy lost by an electron in striking or passing a molecule is a function of the least distance between their paths and is shown to vary inversely as the tenth power of the distance in the case of nitrogen.

3. It is also shown theoretically that this loss of energy should be greatest in the case of heavy complex molecules, less in the case of light simple molecules and very small in the case of monatomic molecules,—thus accounting for the phenomena of elastic and inelastic impacts.

4. From our experimental data, assisted by theoretical considerations, we have been able to calculate the natural frequency of a nitrogen atom with a fair degree of accuracy.

5. There is no such thing as a definite collision between an electron and a gas molecule: the phenomena which have been thus accounted for must be explained as the results of deflections and losses of energy in amounts which depend on the distance between the electron and a molecule at their nearest approach.

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NOTES ON THE ELASTIC PROPERTIES OF PHOSPHOR-BRONZE WIRES.

BY ARNOLD J. OEHLER.

INTRODUCTION.

THE work by Guthe,[1] Guthe and Sieg,[2] and Sieg,[3] on platinum-iridium wires when used as suspensions for torsion pendulums, showed some remarkable elastic properties of that alloy. The principal one of these was the variation of the period with the amplitude of vibration. It was these studies that made it seem very desirable to test other alloys by a similar method, and because of the extensive use of phosphor-bronze wires as delicate suspensions, this work has been devoted to that alloy.

The wires employed in these experiments represented thirteen successive drawings from an original sample and ranged in diameter from .100 mm. to .508 mm. These wires were very kindly supplied by the American Electrical Works of Phillipsdale, Rhode Island.

Some work by Professor Sieg and the writer,[4] in 1914, showed that the periods of torsional vibrations of these wires were not constant but varied widely with the different amplitudes. The problem of this work was to study more carefully, the elastic nature of this alloy.

The apparatus was the same one employed and described by Sieg,[5] and the method of timing the vibrations was also essentially the same as in that work. The length of the suspension most commonly used was 30 cm. and the initial twist was, in most cases, ten degrees per cm. length of the suspension.

The whole problem of the elasticity of this alloy is very complicated and involved and many of the points have been carefully investigated. However, only a few of the most striking points will be discussed in this paper.

[1] K. E. Guthe, Ia. Acad. Sci., 15, p. 147, 1908. Abst. in PHYS. REV., 26, p. 201, 1908.
[2] K. E. Guthe and L. P. Sieg, PHYS. REV., 30, p. 610, 1910.
[3] L. P. Sieg, PHYS. REV., 31, No. 4, p. 421, 1910.
[4] L. P. Sieg and A. J. Oehler, Ia. Acad. Sci., 22, p. 321, 1915.
[5] L. P. Sieg, *loc. cit.*

DISCUSSION OF RESULTS.

The Three States of the Wires.—The first experiments soon showed that there were three distinct states, from one to another of which, the wires would change. This is most readily seen from the three typical period-amplitude curves shown in Fig. 1.

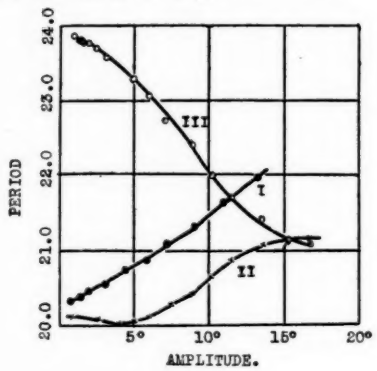

Fig. 1.

Curve *I.* represents state I. in which the period of vibration decreases with the fall in amplitude.

Curve *II.* represents state II. and is similar to curve *I* in the larger amplitudes but makes a departure from that type at an amplitude of about 4 degrees per cm. length of the wire,[1] and from that time on, the period increases as the amplitude decreases.

Curve *III.*, representing state III., shows the greatest departure from a constant period and indicates a continual increase in period as the amplitude decreases.

Of these three states, II. and III. were the most common and state I. was easily changed into state II. There were several cases in which the period was essentially constant below an amplitude of 4 degrees per cm. length of the wire, and it thus appears that type I. is closely connected up with type II. Types II. and III. are very distinct, however, and no evidence has been obtained that would show an intermediate state between the two. When there was change of state the change was always complete from one to the other.

The magnitude of this variation of the period with the amplitude is best seen by considering the curves of Fig. 1 in detail. The three

[1] By amplitude it shall be understood that we mean the complete angular displacement of the pendulum between its extreme positions of vibration. Thus we really have double amplitudes and the above minimum really occurs at an amplitude of about 2 degrees per cm. length of the suspension.

Also the periods mentioned in this paper are, strictly speaking, only half-periods. The method of experimentation made this present use desirable.

curves represent the results from an identical sample of wire (no. 1), which was, however, subjected to various treatments to be described later. During the experiments the load, the length of wire and the moment of inertia of the pendulum were always the same. It is seen that the maximum variation in period is about 3.85 seconds or a variation of 16 per cent. The significance of such a wide variation in the periods of torsional vibrations is very evident. It is also highly probable that the variation is too complicated to be corrected for, in any apparatus in which these suspensions are used.

The Effect of Drawing.—It was soon found that each state was quite stable in itself and unless the wires were subjected to vigorous treatment, the curves for a given sample of wire were always of a definite type. This was especially true of states II. and III. This discovery made it possible to study more carefully, each state.

When the work was undertaken, it was thought desirable to note the effect of drawing on the wires. A sample of each of the seven smallest of the wires was tested, and period-amplitude curves were plotted.

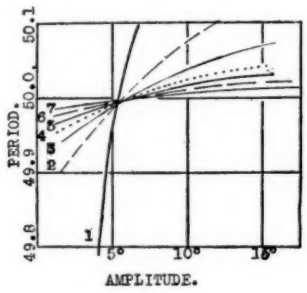

Fig. 2.

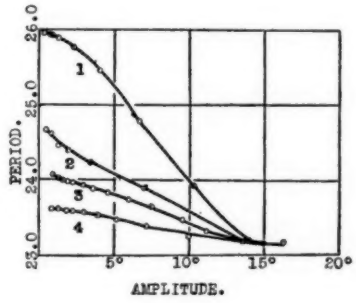

Fig. 3.

Fig. 2 shows these curves reduced to a common period at a chosen amplitude. The wires are arbitrarily numbered, in general inversely as the succession of the drawings. The diameters are given in the table below.

Wire No.	Diameter.
1	.100 mm.
2	.114
3	.133
4	.145
5	.165
6	.181
7	.206

There is seen a tendency for the effect to become more and more marked as the diameter of the wire is drawn smaller. The same effect is better shown in Fig. 3 which represents the curves for the four smallest wires

when these wires were thrown into state III. Here, as in Fig. 2, the curves have all been reduced to a common period at a certain amplitude. An explanation of this has been offered by Sieg,[1] who observed the same effect in platinum-iridium wires.

States II. and III.—The bringing about of certain states in the wires was at first very difficult, because of the apparent inconsistencies in the effects of annealing and artificial vibrations. From the experiments which need not be enumerated here, it became evident that the load supported by the wires, the slowness of the period of vibration, and the length of the suspension, had nothing to do with the state of the wire. But apparently identical treatments of any wire would cause sometimes one state and sometimes another. The wires were heated to different temperatures up to a glow-heat and changes of state occurred seemingly without regularity. The time during which the wire was heated and the rate of cooling after heating, gave no certain clue as to what state to expect.

Similarly, the impressed or artificial vibrations,[2] at the rate of about forty complete vibrations per minute, gave inconsistent changes of state when the duration of the vibration was short. It was noted, however, that if the vibrations were continued long enough, state II. would finally result. Only one exception to this has been found and that was a sample of wire no. 3, which was not changed from state III. in thirteen and one-half hours of continual vibration. The time required to change wire no. 4 from state III. to II. was twelve hours. Wire no. 1 required something over an hour of vibration for the same change. Thus the required time is quite long and depends upon the size and previous history of the wire. From the evidence now at hand, it seems safe to assume that this treatment will always cause state II. as the final state.

It was finally discovered that long continued annealing while the wire supported a small load, caused state III. This was accomplished by sending a moderate current through the wire and maintaining it for the desired time. Wire no. 4 was changed to state III. at every trial by allowing it to carry a current of 1 ampere for twelve hours while it supported a load of 25 grams. The same wire was, however, not changed from state II. to state III. by the same current flowing for 38 hours, when it supported a load of 154 grams.

On one occasion wire no. 1 was changed from state II. to III. simply

[1] L. P. Sieg, PHYS. REV., 35, p. 347, 1912.

[2] This was accomplished by arresting by means of stops, the supported pendulum without relieving the wire of its load, and, by means of gearing and a rack, imparting to the supporting-rod an angular harmonic motion that could be varied in period and amplitude between wide limits.

by allowing it to support a load of 27 grams undisturbed for four months. This process has not been repeated but it is very probable that this treatment is similar in effect to long-continued annealing.

From the data at hand we may say that, in general, (1) long-continued vibrating causes state II. and (2) long-continued annealing at a comparatively high temperature, with the wire supporting a small load, causes state III.

SUMMARY AND CONCLUSION.

The points to be emphasized in this preliminary report, are:

1. There are three distinct classes or states into which these wires can fall.

2. Drawing of the wires has a tendency to increase the effect of a varying period with the amplitude.

3. In general, long-continued vibrating results in a wire's falling into what I have called state II.

4. In general, long-continued annealing results in a wire's falling into state III.

In conclusion we may say that phosphor-bronze wires are not desirable as delicate suspensions, in many cases where they are used. While these peculiarities may be typical of this alloy alone, it is very probable that all alloys have their distinct peculiarities just as platinum-iridium wires and phosphor-bronze wires have been found to have. It thus seems desirable that a careful study be made of all alloys, both from the standpoint of their crystal structure with the resulting elastic constants, and from the standpoint of their practical uses.

In conclusion I wish to acknowledge my indebtedness to the staff of the department of physics for their interest in the problem, and especially to Dr. Sieg for suggesting it, and for his encouragement and assistance during the progress of the work.

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TEMPERATURE AND BLACKENING EFFECTS IN HELICAL TUNGSTEN FILAMENTS.[1]

BY B. E. SHACKELFORD.

IT has frequently been noticed by those working with helical tungsten filaments that the inside of the helix is very much brighter than the outside. A photograph of the projected image of such a filament, that of a commercial Type C Mazda lamp, is shown in Plate I., Fig. 1. It will be noticed that the inside sections, sloping downward toward the right, are much brighter than the outside sections, sloping downward toward the left. The edge of the outside also displays a greatly increased brightness and the very center of the inside of the turns is noticeably darker than the rest of the inside. Some few bright streaks appear on the surface but these are due to scratches and will not be considered in the discussion.

Pyrometer measurements on the outside and on the brightest portions of the inside of the same turns of such filaments show the brightness[2] of the latter to be, in some cases, more than twice that of the former.

The object of the present investigation is to determine the causes of the observed differences. Two possible explanations have been suggested; the interior may be at a temperature sufficiently higher than the exterior, or the increased brightness may be due to the radiation added by reflections within the helix. The two are not mutually exclusive. The only previously published work on the subject is that of Coblentz.[3]

Coblentz, using a "microscope" pyrometer, found the appearance of the edges changing so markedly with focus that he doubted whether the inside were really much brighter. He found "that some parts of the inner surface of the turn had a higher apparent temperature than the outside surface of a turn of the spiral." From these observations and from others made with a spectrobolometer he concluded that "the light coming from within the helix was not appreciably modified or blackened."

[1] Preliminary notice published in Journal Franklin Institute, 180, p. 619, 1915. Some sections have since been checked by Langmuir, PHYS. REV., 7, p. 302, 1916.

[2] Luminous intensity/area.

[3] Elec. World, 64, p. 1048, 1914.

Photographs showing brightness distribution.

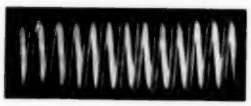


Fig. 1. Commercial filament.

Fig. 2. Special polished filament.

B. E. SHACKELFORD.

Matching the pyrometer filament first against the outside and then against the inside of the helix he found that the same increase of potential difference across the pyrometer lamp was necessary for a match through both red and blue filters. From this he concluded that the increase in brightness was the same as that due solely to increased temperature. The outside matches for red and blue corresponded, however, to different potential differences across the pyrometer lamp, and equal increments of these at different original temperatures do not correspond to equal temperature increments. This is a well-known fact and easily observable experimentally.

Coblentz adds, however, that the phenomenon "is to be considered due in part to black body radiation from the interior," and so arrives at no very definite conclusion as to the relative magnitude of the effects of the two causes of brightness difference. It was thought advisable, therefore, to investigate the question more thoroughly.

TEMPERATURE-BRIGHTNESS RELATIONS.

If we differentiate the Wien equation,

$$J_\lambda = c_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}, \tag{1}$$

we obtain,

$$\frac{dJ_\lambda}{J_\lambda} = \frac{c_2 dT}{\lambda T^2}. \tag{2}$$

Substituting in equation (2) the values $\lambda = 0.66\mu$, $C_2 = 14{,}500$, we find that at a red black body temperature of 2250° K.[1] a 1 per cent. change in brightness is equivalent to 1°.8 change in temperature. As stated above, brightness differences of over 100 per cent. are found on going from the outside to the inside of the coil. Assuming that the whole of this increase is due to temperature, this difference must be of the order of 200°.

Angell[2] gives the following formula for the difference in temperature between the inner and outer surfaces of a hollow cylinder (the limiting case of a helix) heated by the passage of an electric current,

$$T_2 - T_1 = -\frac{EI}{2k}\left[\frac{r_2^2 - r_1^2}{2} - r_1^2 ln \frac{r_2}{r_1}\right] \tag{3}$$

where E = potential difference per unit length,
I = current density,
k = thermal conductivity,
r_2 = external diameter,
r_1 = internal diameter.

[1] This temperature notation is used throughout the present paper.

[2] PHYS. REV., 33, p. 421, 1911.

At a temperature of 2250° K., using Worthing's[1] values, $k = 1.35$ watts/cm. deg., and

$$\frac{EI}{2r_2}(r_2^2 - r_1^2) = 62 \text{ watts/cm.},$$

we find for one of the lamps used ($r_2 = 0.165$ cm., $r_1 = 0.135$ cm.) a temperature difference of 0°.7. This would give the inside an added brightness of less than 0.5 per cent., an amount infinitesimal in comparison with the observed effects. These theoretical considerations seem opposed to the explanation that the difference in brightness is due to difference in temperature.

BRIGHTNESS MEASUREMENTS.

A Holborn-Kurlbaum optical pyrometer employed in this part of the experiment was set up in the manner indicated in Fig. 3, the precautions

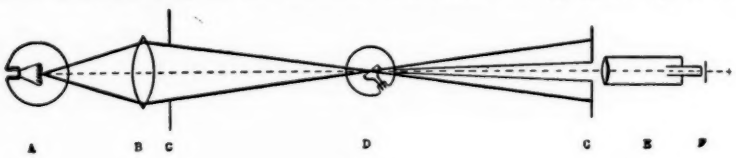


Fig. 3.

Diagram of apparatus. *a*, background lamp; *b*, objective lens; *cc*, diaphragms; *d*, pyrometer lamp; *e*, eyepiece; *f*, monochromatic filter.

mentioned by Worthing and Forsythe[2] being observed. A Zeiss-Tessar lens used for the objective projected the image of the spirals on the 0.05 mm. filament of the vacuum-type tungsten pyrometer lamp. Red and blue filters of effective wave-lengths approximately 0.656μ and 0.493μ were mounted on the eyepiece of the observing telescope, and when necessary the intensity of the background image was reduced by means of sectored discs or neutral tint screens. The currents through the background were maintained at constant values by means of a simple potentiometer arrangement and the currents through the pyrometer lamp were measured with a millivoltmeter across a constant low resistance.

Five gas-filled helical filament tungsten lamps were used, all made from the same wire and wound on the same mandril, the filaments differing from each other only in pitch, which ranged from 1.35 to 2.96 times the diameter of the wire.

At two different temperatures brightness measurements were made on the outside and on the inside of the coils with both red and blue

[1] PHYS. REV., 4, p. 535, 1914.

[2] PHYS. REV., Vol. IV., p. 163, 1914.

filters. The brightest large portions of the inside were utilized, the edges being excluded because of the difficulties attendant upon focusing, and because of the deviations from the cosine law.[1] No difficulties were experienced in this except with the most closely wound filament, and even in this case good readings were possible when a little extra care was exercised in the adjustment.

Table I. gives the average results of many such measurements. In each case the outside-inside brightness ratio is greater for the blue than for the red, *i. e.*, the radiation from the interior of the helix is relatively stronger in the red. If tungsten radiated as a grey body, *i. e.*, if the

TABLE I.

Variation of Brightness Ratio with Pitch.

Pitch.	Outside-Inside Brightness Ratios.					
	2300° K.			1900° K.		
	$\frac{K_1}{\lambda = 0.656\ \mu}$.	$\frac{K_2}{\lambda = 0.493\ \mu}$.	$\frac{K_2}{K_1}$	$\frac{K_1}{\lambda = 0.656\ \mu}$.	$\frac{K_2}{\lambda = 0.493\ \mu}$.	$\frac{K_2}{K_1}$
2.96	.615	.681	1.11	.656	.682	1.04
2.28	.555	.598	1.08	.581	.604	1.04
1.78	.515	.551	1.07			
1.47	.487	.516	1.06			
1.35	.473	.500	1.055	.498	.515	1.035
1.00	.445	.465	1.045	.456	.470	1.03

ordinates of the energy distribution curve bore a constant fractional ratio to those of the energy distribution curve for a black body at the same temperature, this result would mean that the inside is actually cooler, for the curve is shifted toward the long wave-lengths. As this is obviously contrary to fact, tungsten must radiate more strongly, proportionally, in the blue than in the red, as has been observed previously, and the radiation from the inside must be blackened by repeated reflections. Due to a higher temperature inside of the spiral the distribution should show a shift toward the short wave-lengths and as the opposite is found to be the case, at least the greater part of the increased brightness must be due to reflections rather than to a temperature difference.

In justification of these conclusions it was found that when the brightness ratios were plotted against pitch (Figs. 4 and 5) straight lines were obtained, which when extrapolated backward to pitch unity, corresponding to a closed cylinder or black body, gave limiting values, at a red black body temperature of 2300° K., of 0.445 for $\lambda = 0.656\mu$ and

[1] A. G. Worthing, Astrophysical Journal, 36, p. 345, 1912.

0.465 for $\lambda = 0.493\mu$. At 1900° K. the ratios became for the former, 0.456 and for the latter, 0.470. These values correspond very closely

Brightness ratios plotted against pitch, in order to determine the emissivity.

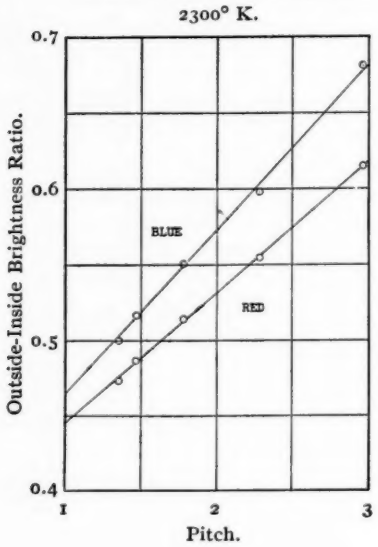


Fig. 4.

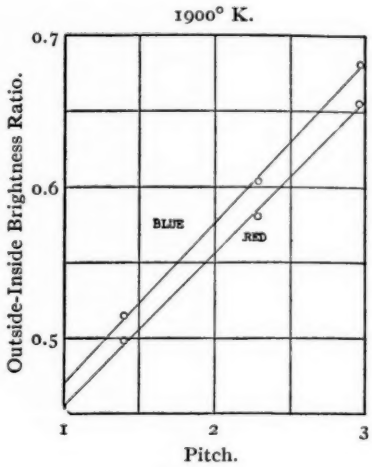


Fig. 5.

with those of Worthing[1] for the emissivity[2] of tungsten, found by an entirely different method.

THE POLISHED FLATTENED FILAMENT.

A piece of 0.5 mm. tungsten wire was flattened to about half its original thickness, after which it was polished, coiled with a pitch equal approximately to twice the width of the ribbon and mounted as a lamp. The characteristics of this filament are in general the same as those of the commercial ones, the differences being those of degree (Plate I., Fig. 2). The middle dark portion of the inside is much broader and more pronounced, as are also the two adjacent bright portions. In addition to these there may be seen on the rounded edges other light and dark bands. This photograph alone is almost conclusive evidence that the phenomenon is one of reflection. The central sections of the turns appear dark since they are viewed normally and there is no opportunity for light to be added by specular reflection. Despite the fact that the surface is not an optical one there is very little diffuse reflection, as evidenced by the fact that the brightness of this part is only slightly greater than that of

[1] PHYS. REV., VII., p. 497, 1916.

[2] Relative to that of a black body.

the outside. The two central bright bands are seen to be images of the adjacent filaments, which fact accounts for the added brightness. The other bright bands are the images of the next succeeding turns of the helix, and the dark bands correspond to the spaces between the turns.

Quantitative support of these conclusions is obtained from measurements made at 2000° K. as above. For this lamp also the maximum brightness of the inside was found to be of the order of twice that of the outside. The broad dark bands in the center of the interior, however, showed a brightness only slightly greater than that of the exterior (Table II.).

TABLE II.

Polished Filament, Red Black Body Temperature 2000° K.

Number of Turn.	Outside-Inside Brightness Ratio.		$\frac{K_2}{K_1}$
	$\frac{K_1}{\lambda = 0.656\mu}$	$\frac{K_2}{\lambda = 0.493\mu}$	
1	.942	.977	1.04
2	.970	.980	1.01
3	.918	.917	1.00
4	.973	.967	0.99
Average.....	.....	.....	1.01

Although some of the four turns measured exhibited a more perfect polish than did others, still in every case the brightness of the outside approached very closely to that of the central portion of the inside, and on the average even this portion of the inside was redder, as in the cases mentioned above. For the second turn, assuming the whole brightness difference to be a temperature effect, we obtain in the manner indicated above a maximum allowable temperature difference of about 5 degrees between the outside and the dark band of the inside. For the brighter portions of the inside of the coil this difference would amount to approximately 175 degrees. It would be impossible for these to have a temperature 170 degrees higher than the middle of the same turn. The brightness difference has been practically entirely localized, simply by the polishing of the filament. This obviously could not be true of a temperature effect, and reflections within the helix must be at least the principal cause of the brightness difference.

COLOR MATCHING.

Photographs of the helical filaments were made and cut as templates to transmit only the radiation from the interior and exterior respectively. These were placed in turn against a ground glass diffusing screen, upon which the image of the helix was then projected. By means of a Lummer-

Brodhun photometer these filaments, operated at 2000° K., were color matched against a comparison lamp. When the comparison lamp was adjusted to match the radiation from one of the surfaces, and then that from the other was permitted to pass into the photometer, the difference in color was very evident.

As will be seen from Table III., in order to match the outside, 0.5 per cent. greater current was necessary through the comparison lamp than to match the total radiation. Two per cent. less current was required to match the interior radiation, the lamp distances being adjusted in each case so as to give equal brightness contrast on the photometer screen. Again it is shown that the interior is redder than the exterior, so that the major portion of the increased brightness must be due to reflected light.

TABLE III.

Color Matching. Temperature 2000° K.

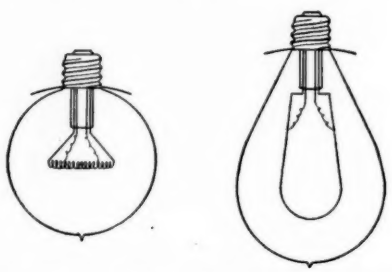
Current, in amperes, through the comparison lamp necessary to match the helix.

Setting.	Total Radiation.	Outside of Helix.	Inside of Helix.
1	2.243	2.247	2.222
2	2.232	2.240	2.182
3	2.227	2.227	2.200
Average.....	2.23	2.24	2.20

The various settings were made on different days and with slightly different currents through the helix.

RESISTANCE MEASUREMENTS.

Two filaments, one a hairpin, the other a helix, were made from the same piece of wire, provided with potential leads approximately a quarter of the way from the ends, and mounted in lamp bulbs (Fig. 6). Owing



Coiled and hairpin filament lamps used for the temperature-resistance measurements.

Fig. 6.

to the size (0.05 mm.) of these leads it was found inadvisable to weld them on, and they were tied tightly around the filament. The temperatures

of the hairpin filament and of the outside of the helical filament were found to be very uniform over the lengths used. These temperatures were measured with the optical pyrometer and the resistances were measured by comparing the fall of potential across the lamps with that across a known resistance. The results are given in Table IV. and

TABLE IV.

Values of the Resistances of the Helical and Hairpin Filaments at Various Temperatures.

A—Helical Filament.		*B*—Hairpin Filament.	
Temperature.	Resistance.	Temperature.	Resistance.
1360	49.6	1390	48.9
1372	49.4	1413	48.5
1473	55.4	1519	55.3
1510	57.5	1524	55.4
1558	60.1	1650	61.2
1586	60.8	1695	63.0
1589	60.9	1793	67.9
1605	62.5	1795	68.2
1635	64.2	1885	72.5
1732	68.9	1948	75.8
1778	70.5		
1795	71.7		
1798	72.3		
1908	77.4		
1987	81.8		

shown diagrammatically in Fig. 7, the line *A* representing the helical and the line *B* the hairpin filament. When the ordinates of *B* are multiplied by the ratio of the two ordinates at 1400° K., in order to bring the filaments to equivalent lengths, the new curve for *B* coincides exactly with *A* as drawn. Therefore, within the limits of experimental error the two filaments have the same temperature coefficient of resistance. This would not be the case if the helix heated unduly on the inside, for then the average temperature would be greater than that measured by the pyrometer, the effective resistance would be greater, and the curve for the helix would be steeper. From resistance measurements, therefore, the temperature throughout the helical filament is sensibly that of the outside.

SUMMARY.

To summarize: The data show:

1. That the interior of the helical tungsten filament has a maximum brightness of the order of twice that of the exterior.

2. That the inside is redder, and that the increased brightness is, in the main, due to internal reflections.

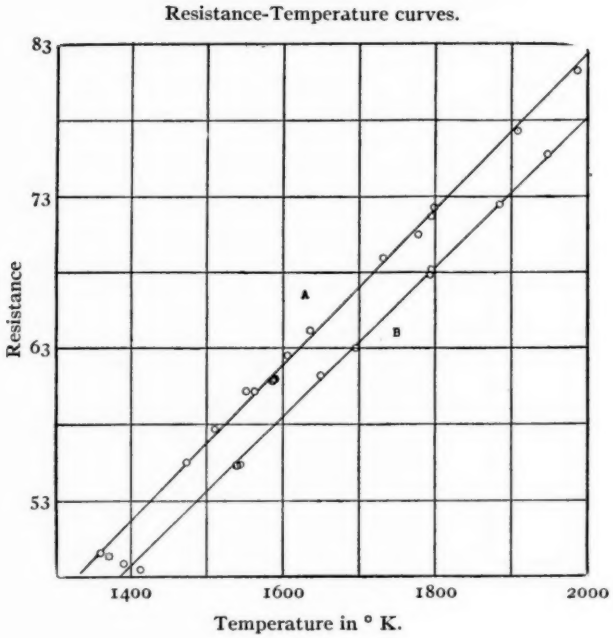


Fig. 7.

3. That the temperature difference between the inside and the outside of an unusually heavy filament operated at 2000° K. is not greater than 5°, as the maximum difference allowable from resistance and pyrometer measurements.

4. That for all ordinary lamps the calculated difference is of the order of 1°.

5. Values of the emissivity of tungsten have been obtained for two wave-lengths at two temperatures.

6. That tungsten radiates selectively in the visible portion of the spectrum in such a way as to make the amount of radiation, relative to that of a black body, greater in the blue than in the red.

I desire to acknowledge my indebtedness to Director E. P. Hyde, of the Nela Research Laboratory, for suggesting the problem and placing the facilities of the laboratory at my disposal while Brush Research Fellow; and to Dr. A. G. Worthing for constant help and criticism.

NELA RESEARCH LABORATORY,
CLEVELAND, OHIO,
June, 1916.

THE COEFFICIENT OF VISCOSITY OF AIR BY THE CAPILLARY TUBE METHOD.

BY EFFIE MARKWELL.

A GREAT deal of work has been done on the determination of the coefficient of viscosity of air by the capillary tube method, especially at the University of Halle. Mr. I. M. Rapp,[1] at the University of Chicago, in 1913, made a determination with extreme care, using improvements in the method, and obtained as a mean result from work on four carefully chosen tubes a value of 1.4 per cent. lower than that found by the Halle observers but in close agreement with the mean taken by Professor Millikan[2] as the most probable value of η and estimated by him as correct to one-tenth per cent. Recently, however, Mr. A. Gille,[3] of the University of Halle, using one tube only, secured a value .6 per cent. lower than previous determinations at Halle but still about .8 per cent. higher than that obtained by Rapp. His value, however, is in perfect accord with Vogel's[4] estimate of the most probable value. The fact that all constant sources of error such as ellipticity and non-uniformity of bore, which are unavoidable by the capillary tube method, operate to increase the apparent value of η is a strong reason for giving greater weight to low values of η obtained by the capillary tube method than to high ones, especially if the low values are found as in Rapp's experiment with larger tubes than those used in obtaining the high values. In view, however, of the papers of Messrs. Gille and Vogel it was thought worth while to make another determination with Rapp's apparatus but using new tubes.

A large number of tubes were examined at the ends for ellipticity with a micrometer microscope and tested for non-uniformity with a thread of mercury. The ones selected were calibrated by Fisher's method.[5] The measurements of the lengths of the column of mercury in the different positions were made on an accurately calibrated dividing engine. Such precautions were taken that the error in weighing the mercury was

[1] PHYS. REV., vol. 2, p. 363, 1913.
[2] Annalen der Physik, vol. 41, p. 759, 1913.
[3] Ann. d. Physik, p. 799, Dec., 1915.
[4] Ann. d. Physik, vol. 43, p. 1235, 1914.
[5] PHYS. REV., vol. 28, p. 673, 1909.

estimated to be always less than one tenth milligram and the smallest amount weighed was about 2.2 grams. Table I. shows the length and radius of each tube used.

TABLE I.

Length and Radius of Tubes.

No.	Length.	Radius.
1	68.257	.031798
2	91.19	.047366
3	87.756	.039179

The capillaries were carefully cleaned with a solution of sulphuric acid and potassium bichromate and distilled water and thoroughly dried before calibrating and again before using.

The apparatus used was essentially the same as Rapp used and the arrangement of the different parts may be seen from the diagram. Air from the laboratory pressure system, dried and freed from dust by passing through calcium chloride and phosphorus pentoxide tubes with filter plugs of glass wool, enters A, then passes through the coils CC to the capillary D. A is open to the air through B_1 and B_2. The pressure of the air entering CC, therefore, adjusts itself to atmospheric pressure. However, an oil manometer was inserted at A so that if for any reason

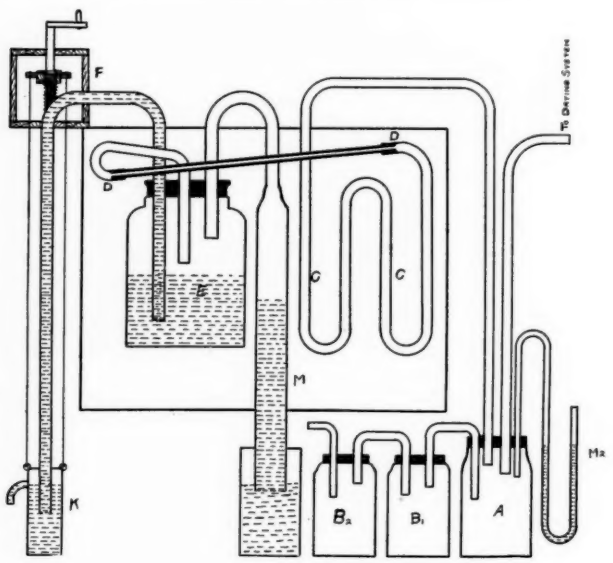


Fig. 1.

the pressure did not so adjust itself it could be observed and corrected. B_1 and B_2 contain phosphorus pentoxide and glass wool so as to prevent any possible diffusion of dust or moisture back into A. The tubing

used in CC was so large as to produce no appreciable pressure gradient and the part in the bath was about 300 cm. long so that the air was sure to be of the same temperature as the bath before entering the capillary. The bath held about a cubic meter of water and was thoroughly stirred by a motor-driven fan stirrer in the bottom. The temperature of the bath was easily held constant to within .05° C. for a series of runs without recourse to a thermostat and we were not able to detect any differences of temperature in different parts of the bath after stirring.

As oil escapes from the reservoir E through the siphon tube H, p_2, the pressure on the air leaving the capillary is reduced below atmospheric pressure and the driving pressure, $p_1 - p_2$, is measured directly by the manometer M and is equal to hgd_{oil}. At first a small tube was used in the manometer but the correction for capillarity was found to be large and as it was difficult to measure this with sufficient accuracy, a tube about 2.5 cm. in diameter was substituted so that no correction was necessary. The lower end of the siphon tube extended into a bucket K which was suspended from a micrometer F. As air was admitted into E through the capillary, K was lowered with the micrometer at such a rate as to keep the height h of the liquid in the manometer constant. Polarine oil which has a low vapor tension and density was used in the reservoir and manometer. This enabled us to read low pressures with much greater accuracy than if mercury had been used in the manometer. The siphon tube and the manometer were necessarily at room temperature while the rest of the apparatus was at the temperature of the bath, however the readings were nearly all taken when the temperature of the bath was within 1° C. of the temperature of the room. Since the oil was found to be slow in coming to the temperature of the bath no readings were taken until the oil had been in the bath several hours, usually more than twelve hours. The volume of air at p_2 was determined by collecting and weighing the oil which flowed out of the bucket. Since lowering the bucket caused less oil to flow out this was corrected by adding to it the weight of a cylinder of oil whose diameter was the outer diameter of the siphon tube and whose height was equal to the distance through which the bucket had been lowered. Weighings were made to .01 gram and the smallest amount weighed was more than six hundred grams. The time was measured with an accurately calibrated stop watch which was frequently compared with the standard clock. The density of the oil used was tested at several temperatures between 15° C. and 30° C. and a line drawn through these points from which the density at the required temperature was read.

By virtue of the relations:

$p_1 - p_2 = hgd_{oil}$, $p_1 = Bgd_{Hg}$ and $V_2 = \frac{M}{d_{oil}}$ Poiseuille's law, viz.,

$$\eta = \frac{\pi R^4 (p_1 - p_2)^2 t}{16 L V_2 p_2} \tag{1}$$

reduces to

$$\eta = \frac{\pi R^4 d^2_{oil} ght \left\{ 2B - h\frac{d_{oil}}{d_{Hg}} \right\}}{16ML \left\{ B - h\frac{d_{oil}}{d_{Hg}} \right\}}. \tag{2}$$

TABLE II.

Data for One Series.

l = 68.257 cm.	t_B = 22.4.
R = .031798 cm.	d_{oil} = 8947.
g = 980.3.	d_{Hg} = 13.596.
B_{cor} = 74.3114.	$h_{d\ oil}$ = 46.2 cm.
t_R = 22.2–22.35.	$h_{d\ Hg}$ = 3.0402.

Area of cross section of siphon tube .73757 sq. cm.

No.	K Lowered.	Mass.	$\text{Mass}_{cor.}$	Time.	$\eta\ 10^{-7}$
13	3.2 cm.	710.09g.	712.40g.	600 sec.	1,834.3
14	3.2	710.79	712.90	"	1,833.0
15	3.5	710.48	712.79	"	1,833.25
			712.7		1,833.5

Corrected for slip $\eta' = 1{,}835.4 \times 10^{-7}$
Corrected for end effects $\eta'' = 1{,}825.2 \times 10^{-7}$
Reduced to 23° C. $\eta_{23} = 1{,}828.2 \times 10^{-7}$

The values obtained by the formula (2) were corrected for slip by

$$\eta' = \eta \left(1 - \frac{4\zeta}{R} \right), \tag{3}$$

where $\zeta = 0.000083$. There are also departures from Poiseuille's law due to the energy in the stream lines at the ends of the tube. M. Brillouin and Fisher have both worked out a formula for this correction and Rapp has shown that both give the same results when applied to his data. Rapp also derived an empirical formula equivalent to Fisher's theoretical formula. This empirical formula

$$\eta'' = \eta' \left(1 - A \frac{R^4 (p_1 - p_2)}{1} \right) \tag{4}$$

was used in correcting values obtained from (5) and finally the results were reduced to a common temperature of 23° C. by the equation

$$\eta_{23} = \eta_t''(1 + .00276\{23 - t\}). \tag{5}$$

Table III. gives the values obtained for each series of runs with each of the capillaries and Table IV. gives the mean value for each tube reduced to a common temperature, the per cent. of probable error for each and the mean value for the three tubes.

The mean value obtained from two of the capillaries agrees within

TABLE III.

Data for Tube No. I.

Series No.	Temp.	$p_1 - p_2$ cm.	$\eta \times 10^{-7}$	$\eta' \times 10^{-7}$	$\eta'' \times 10^{-7}$	$\eta_{22} \times 10^{-7}$
1	22.9	18.85	1,842.5	1,843.8	1,828.03	1,828.5
2	22.9	16.65	1,838.7	1,839.9	1,826.1	1,826.6
3	22.	17.65	1,835.5	1,836.8	1,822.1	1,827.1
4	22.	15.4	1,838.9	1,840.2	1,827.3	1,832.3
5	21.6	17.85	1,840.8	1,842.1	1,827.2	1,834.3
6	21.9	17.90	1,839.7	1,841.0	1,826.0	1,831.5
7	21.6	15.30	1,837.7	1,839.0	1,825.3	1,832.4
8	21.9	16.80	1,835.5	1,836.8	1,822.8	1,828.3
					Mean	1,830.1

Data for Tube No. II.

1	23.05	47.20	1,828.3	1,830.2	1,819.8	1,819.6
2	23.00	41.60	1,832.9	1,834.8	1,825.6	1,825.6
3	22.4	46.20	1,833.5	1,835.4	1,825.2	1,828.2
4	22.4	40.30	1,830.7	1,832.6	1,823.6	1,826.7
5	23.55	46.25	1,837.3	1,837.8	1,828.6	1,826.2
6	23.5	47.40	1,834.8	1,836.7	1,826.2	1,823.7
7	22.23	46.00	1,834.0	1,835.9	1,825.9	1,829.8
8	22.23	41.35	1,832.5	1,834.4	1,825.2	1,829.0
9	21.8	45.40	1,825.9	1,827.8	1,817.8	1,823.8
10	21.8	39.80	1,827.0	1,828.9	1,820.1	1,826.1
					Mean	1,825.9

Data for Tube No. III.

1	23.45	32.10	1,836.4	1,837.95	1,824.9	1,822.6
2	21.9	30.45	1,835.1	1,836.7	1,824.3	1,829.8
3	22.3	31.05	1,830.7	1,831.7	1,819.1	1,822.6
4	22.05	30.85	1,828.5	1,830.0	1,817.5	1,822.3
5	22.05	28.65	1,833.2	1,834.7	1,823.1	1,827.9
6	22.7	31.60	1,832.3	1,833.9	1,821.3	1,822.8
7	24.0	33.20	1,840.9	1,842.5	1,828.9	1,823.9
8	22.6	31.18	1,834.3	1,835.8	1,823.2	1,825.2
9	21.1	30.40	1,835.9	1,837.5	1,825.1	1,829.6
10	22.05	30.67	1,835.6	1,837.1	1,824.7	1,829.5
11	22.1	30.45	1,832.5	1,834.1	1,821.8	1,826.3
12	22.2	30.50	1,835.0	1,836.6	1,824.2	1,828.2
					Mean	1,825.8

TABLE IV.

Tube No.	7×10^{-7}	Per Cent. of Probable Error.
1	1,830.1	.044
2	1,825.9	.033
3	1,825.9	.037
	Mean 1,827.3	

one tenth per cent. with the value taken by Professor Millikan, namely, $\eta_{23} = .00018240$ while the other is three tenths per cent. higher than his value. Since the consistency of the results obtained with any given tube shows a probable error of less than .05 per cent. and since all irregularities in the bore of the tube tend to push up the apparent value of η it must be inferred that the bore of tube no. I. is a less perfect cylinder than are the bores of the other two.

These experiments indicate, then, that with sufficient care in the selection of the tubes the capillary tube method is capable of yielding results on η which are in good agreement with those obtained by other methods and that the value taken by Professor Millikan, viz., $\eta_{23} = .0001824$ can not be more than one tenth per cent. too low.. Careful work by this method which yields values of η essentially higher than this is presumably to be interpreted as indicating imperfections in the cylindricity of the bore of the capillary.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
June 7, 1916.

THE ECHELETTE GRATING.

By A. Trowbridge.

IN a paper by Mr. I. B. Crandall and myself published in the Philosophical Magazine in October, 1911, an account was given of an investigation of the optical properties of a number of gratings the rulings of which were of an approximately triangular form. These gratings were prepared in the manner described by R. W. Wood[1] and two of those investigated had been ruled by Professor Wood on one of the Rowland engines.

This type of grating, designated "echelette" by Wood, has hitherto been ruled only in the soft metal gold in which the groove is pressed rather than cut by the tool. However some of the speculum gratings cut by Rowland and others show the property which Wood's echelette gratings show in a more marked degree.

This property is: The concentration of energy into one or more of the spectra on one side of the central image with the consequent abstraction of energy of some wave-lengths from other spectra or from the central image.

As a result of this, when the spectra of a distant light source are viewed, the central image appears colored and generally weak and the color changes as the angle at which the central image is formed is made to vary by changing the angle of incidence.

The low order spectra appear with slightly varying intensities on both sides of the central image but with an unsymmetrical triangular groove there appears on one side a broad band of white light in which the greater part of the incident energy is concentrated. This was called by Wood the "oblique image" and consists in the diffraction images of the first class formed by the single elements of the grating.

If all of the original surface on which the grating was ruled were destroyed there would be, strictly speaking, no central image though there would be energy diffracted in the direction in which the central image would appear.

When a very small portion of the original surface of the metal is left the energy in the central image will be in part that for which the path

[1] Phil. Mag., 6, vol. 20.

retardation over the element of the undisturbed surface is zero and in part that for which the retardation over the element of the new surface is some odd number (other than one) of half-waves. This explains why in general the central image is colored and its color depends on the angle of incidence.

The gratings studied by Wood,[1] Wood and Trowbridge[1] and Trowbridge and Crandall[2] were squeezed on gold-plated copper plates which were far from flat surfaces though for the infra-red energy with which their behavior was studied they were less imperfect than they appeared to be to the eye.

It has seemed to the writer worth while to attempt to rule on really flat speculum plates gratings which should retain the property of energy concentration exhibited by the rather imperfect echelette gratings hitherto constructed. As it seemed out of the question to squeeze the grooves in so hard a metal as speculum it was necessary to grind a diamond cutting tool which would leave the cut with a polish comparable to that of the original surface.

The engine with which the gratings described further on were ruled was one constructed in the shop of the Palmer Physical Laboratory under the direction of the late Professor C. F. Brackett. Although it was completed, tested and adjusted by Professor Brackett it has not been described and so for this reason a brief account of those features in which it differs from the well-known engine of Rowland is inserted in this place.

Three steel screws, two right-handed and the other left-handed were cut with a pitch of about one millimeter and the ends of the axes were case-hardened. After a rough lapping with lead the two right-handed screws were ground with a three-piece nut whose length was approximately three quarters the length of the screw. During this grinding the screw stood vertically in a small cylinder containing oil and abrasive and the weight of the nut was counterpoised. The cylinder in which the grinding took place was surrounded by a water-bath with thermostat and the grinding was arranged to reverse automatically when the nut came flush with either end of the screw.

From time to time the two right-handed screws were " log-rolled " on the left-handed screw to correct the error of run. This was done in a frame which supported the left-handed screw in bearings while the two right-handed screws meshed the threads of the left-handed screw and were in part supported by it and in part by vertical ways against which the case-hardened axes of the screws rested. The left-handed screw was

[1] Phil. Mag., 1910.

[2] *Ibid.*, 1911.

driven by a motor which reversed when either of the right-handed screws had crept through a certain distance in the direction of its axis. Very fine abrasive was used in this operation and the effect was not only to correct the error of run but also to keep the form of the thread sharply triangular.

In place of the ratchet device for advancing the screw adopted by Rowland a gear wheel driven by a worm was mounted on the axis of the screw. This gear wheel was made in three sections; one section formed part of the casting of the hub and the other two sections were rings. The three sections were held together by twelve bolts and nuts and the whole was cut in the ordinary manner into a gear wheel by means of a cutter of sixteen flutes, five of which were cutting at one time.

After a reasonably good gear wheel had been cut one of the sections was changed in azimuth with respect to the other two by unbolting and resetting the twelve bolts and the cutting was repeated. In this manner over one hundred different arrangements of the sections of the gear wheel were made and it was judged probable that all systematic error had been removed. Equal care was not necessary in making the worm as the engine was designed to rule only once for each complete revolution of the worm. The thrust bearings against which the screw bore were optically flat quartz plates mounted with their planes vertical. The ends of the screw were countersunk on the axis and internally ground while they were turned on case-hardened collars which had been previously externally ground while the screw turned between the centers on which it was cut.

Into each of these ground counter-sunk ends of the screw an accurately figured steel ball was placed and these balls thrust against the quartz plates previously mentioned.

The nut fastened to the grating carriage contained sixteen inserts of osage orange wood and it was made in two pieces which were held together by steel springs. The nut was about two inches long and could be rocked automatically to correct for periodic error.

The number of rulings per inch was 8,828 or by use of gears any submultiple of this number. The possible area of ruled surface was about 200 square centimeters.

As may be seen from the above very brief description the chief difference from the Rowland engine consists in the use of an advance by worm and gear instead of by ratchet wheel.

Professor Brackett devoted a great deal of time and energy to making and examining gratings but he left no notes. As he generally refigured the speculum plates on which his trial gratings were cut there remain

very few of his gratings on which to base an opinion of the performance of the engine. From a superficial examination of the few gratings left by Professor Brackett I should say that the device for correcting for periodic error was either inadequate or had been incorrectly set when the gratings in question were ruled.

While the engine in its present state of adjustment is not capable of ruling high grade gratings for use in the visible or ultra-violet region it is in every way satisfactory for making gratings for use in the infra-red.

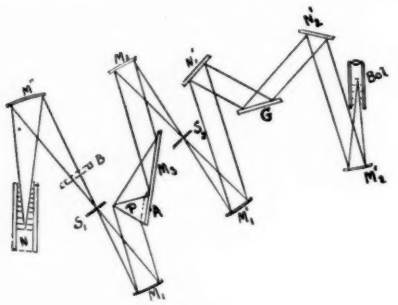


Fig. 1.

The general arrangement of apparatus for the study of the energy distribution in the various orders of the spectra from any grating is shown in Fig. 1.

S_1PS_2 is a prism spectrometer with the usual mirror attachment M_3 due to Wadsworth, which insures that the energy in focus on the slit S' shall have passed through the prism in minimum deviation as the prism is rotated about a vertical axis through A.

S_2GB is a grating spectrometer, the concave mirrors M_1' and M_2' of which are fixed. As the grating G is rotated about a vertical axis in its own plane the various orders may be brought to pass in focus across the plane of the bolometer strip (shown in the enclosure marked Bol.).

N is the filament of a Nernst glower, a real image of which is thrown on the slit S_1 by means of the concave mirror M''. Both the glower and the bolometer were enclosed in water-jacketed housings provided with suitable diaphragms.

In the path of the beam of light a double-walled metal shutter B coated with lamp black could be raised and lowered from the observing table. From the same point rotation of the prism or of the grating could be effected by means of worm and gear mechanisms.

The arrangement chosen for the grating spectrometer, *i. e.*, fixed collimators, is experimentally the most simple as a fixed bolometer is practically essential for sensibility and steadiness of zero.

It is in one way unfortunate however as it involves a change in the angles both of incidence and diffraction in any series of settings and renders it impossible to study the energy distribution in all of the orders at once for any given angle of incidence. However this is not a vital objection to this arrangement as the greater part of the energy is distributed in one or two orders on one side of the central image in the case of an echelette grating and calculation shows that there is not much change in the distribution of energy as the angle of incidence is varied within moderate limits if the wave-length is long. Two different prism spectrometers were used in the work to be described; the larger had a divided circle reading to seconds of arc and concave mirrors of focal length 75 cm. and aperture 13 cm. The prism was a very large one of clear rock salt polished by Brashear. It was 8 cm. high (area of face 100 sq. cm.) and the angle was 60° 01′.

As will be seen from the dimensions of the prism and mirrors the system was designed to give good definition and resolving power but was rather wasteful of energy. Also the great length of the light path in air in going from the Nernst filament to the slit S_2 made the absorption due to water vapor and carbon dioxide in the air so prominent as to badly cut up the energy distribution curves which were being studied. For this reason in the latter part of the work a short focus prism spectrometer made by Schmidt and Haensch was used as thus the air path could be reduced to about one quarter of that of the larger instrument. There resulted from this change both an increase in available energy and a greater smoothness in the energy curves but this gain was naturally at the price of the purity of the spectrum.

The spectrometer S_2GB was the vacuum instrument which has been already described[1] but slightly modified by the use of the two plane mirrors $N_1'N_2'$ inserted to reduce the aberration of the concave mirrors $M_1'M_2'$ by permitting them to be used at nearly normal incidence.

The bolometer strips were one half millimeter wide and 25 mm. long and the galvanometer was one of the D'Arsonval type constructed by Leeds and Northrup and which had a voltage sensibility of 12.5 divisions per micro volt for a scale distance of one meter. The zero of the vacuum spectrobolometer was so steady that the spot of light would stay on the scale for days at a time and therefore the scale distance could be increased to about four meters. This gave ample sensibility for the work in hand the only drawback being that the period of the galvanometer was too long (7 sec.) to permit of rapid work.

The reproducibility of results was quite satisfactory as will be seen

[1] A. Trowbridge, Phil. Mag., 1910.

from plate 3 (curves for 1.7μ and 3.66μ) where the circles represent observations taken one day and the crosses represent observations taken two days later after both spectrometers had been reset on the former readings and the current in the Nernst filament had been adjusted to its old value.

When the long focus, large prism spectrometer was used the purity of the spectrum was such that the wave-length interval occupied by the slit of the grating spectrometer was 0.04μ at $\lambda = 1\mu$, 0.2μ at $\lambda = 3\mu$, 0.1μ at $\lambda = 5\mu$ and 0.08μ at $\lambda = 7\mu$. With the short focus prism spectrometer the purity was about one half as good.

The method by which the energy distribution curves for the various gratings were obtained was as follows: A given wave-length interval in the prismatic spectrum was brought on the slit of the grating spectrometer and the grating was turned so that the central image and the various orders were brought successively on the bolometer strip and the deflections of the galvanometer noted for each setting. Since it had been previously ascertained that the galvanometer deflections were proportional to the energy falling on the strip, curves representing the energy diffracted under any given angle could be constructed from the observed data. An example of such energy curves is given in Plate 3.

In all, three large (nos. 3, 4, 5) and two small (nos. 1 and 2) gratings were ruled and investigated in both the visible and infra-red region of the spectrum. In all five the form of the groove was triangular but the depth of the cut and the inclination of its long side to the original surface was different for the different gratings.

So far as the energy distribution in the infra-red is concerned it will be sufficient to give a graphical representation for one of the gratings and describe the behavior of the others in terms of the differences between them and the example given.

In the paper by Trowbridge and Crandall[1] a formula was deduced for the retardation over the element of an echelette grating on the assumption that the element was triangular in form with one side of the groove making an angle α, and the other side making an angle of 90° with the undisturbed surface of the plate on which the grating was cut. In this formula, which follows, 2θ is the angle between the collimators of the grating spectrometer, *i. e.*, the angle $N_1'GN_2'$ of Fig. 1 and δ is the angle through which the grating has been turned to its given position from the position in which the central image fell on the bolometer strip. In the formula b represents the projection of the inclined side of the cut on the plane of the original surface (thus b equals the

[1] *Loc. cit.*

grating constant in case none of the original surface is left). In the case of most of the gratings studied b was very nearly equal to the grating constant $g = 0.0011508$ cm.

The expression for the retardation over the element is:

$$\rho = 2b \cos\theta \sin(\alpha - \delta) \frac{\cos(\theta + \delta)}{\cos(\theta + \delta - \alpha)}. \qquad \text{(Eq. A)}$$

and in it the quotient of the cosine terms takes account of the fact that for some settings the vertical edge of the groove shadows a portion of the inclined edge over which the retardation is calculated.

It is evident that the retardation vanishes when the grating is turned so that the "oblique image" is on the bolometer strip for then $\delta = \alpha$ and that when the central image is on the strip ($\delta = 0$) the retardation is

$$\rho_0 = 2b \frac{\cos^2\theta \sin\alpha}{\cos(\theta - \alpha)}.$$

This of course cannot be zero if as in practice $\theta < 90°$ and it may be some whole number of waves of some wave-length λ. Energy of this wave-length should be wholly lacking in the central image if $b = g$ and in any case there should be a minimum in the curve of energy distribution in the central image at any wave-length such that ρ_0 is any whole number of waves.

All the gratings show this phenomenon both with visible light when there are many full waves retardation and also with infra-red energy when it is due to one or two waves retardation of the greater wave-length.

The energy distribution in the central image of grating no. 5 (curve a) is shown in Fig. 2 in which the abscissæ represent wave-lengths in μ and the ordinates energy in arbitrary units. Curve b represents the energy reflected from a speculum mirror similar in size and polish to that on which the grating no. 5 was cut. The ordinates for curve b are on a scale 1 : 30 of those of curve a in the wave-length interval from $\lambda = 1\mu$ to $\lambda = 6.2\mu$ and beyond $\lambda = 6.2\mu$ on a scale of 1 : 10.

It will be noted that while the energy maximum in the light reflected from the speculum mirror lies in the neighborhood of $\lambda = 2\mu$ the energy maximum in the central image of the echelette grating lies close to $\lambda = 5.5\mu$ and that there is a very marked minimum at $\lambda = 3.2\mu$ no trace of which is present in curve b and which is therefore undoubtedly due to interference. The slight minimum at $\lambda = 6.9\mu$ is present in both curves and is therefore probably attributable to either atmospheric absorption or to some selective reflection from the speculum metal. The latter explanation is the more probable as the data given in Fig. 2 were obtained with the small prism spectrometer and the vacuum grating

spectrometer and for this reason the probability of atmospheric absorption is small.

Although the authors do not call attention to it, Hagen and Rubens[1] report data on the reflection of speculum metal which show a slight minimum of from 1–2 per cent. in the reflection curve at $\lambda = 7\mu$. As

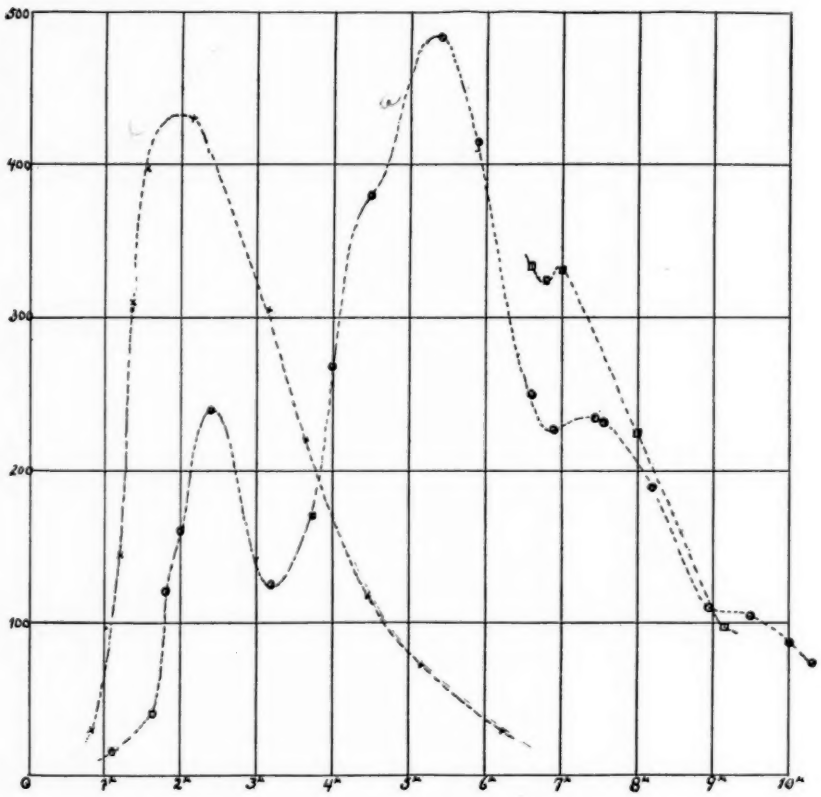


Fig. 2.

both the position and the magnitude of the depressions in curves *a* and *b* correspond with the data of Hagen and Rubens it seems most reasonable to attribute them to a slight selective reflection of speculum metal.

Hagen and Rubens's paper throws no light on the question as to whether the depression at $\lambda = 9\mu$ is due to the same cause since Hagen and Rubens took no observations at or near this wave-length.

In the case of the other gratings the minimum due to interference appears at different wave-lengths on account of the fact that both the depth of the cut and the angle of the inclined surface is different. Thus

[1] Ann. der Physik, 4, 11, 1903, p. 882.

for grating no. 1 the minimum was at $\lambda = 2\mu$, for no. 2 at $\lambda = 2.4\mu$ for no. 3 at $\lambda = 2.8\mu$.[1]

In the case of grating no. 5 the general optical and infra-red behavior was in agreement with the conclusion that the interference minimum at $\lambda = 3.2\mu$ in the central image was due to there being a retardation of one wave, but in the case of grating no. 3 the general optical behavior was only to be reconciled with there being a retardation of two waves of length $\lambda = 2.8\mu$ though there was no evidence of interference due to a retardation of one wave of length $\lambda = 5.6\mu$.

All of the minima noted in the central images of the various gratings were found to lie in the spectral region between $\lambda = 2.0\mu$ and $\lambda = 3.2\mu$, *i. e.*, near the energy maximum of the prismatic spectrum of the source of light. It seems probable that the failure to find minima due to lower orders of interference in the cases mentioned was due to there having been insufficient energy for detecting the effect in the region where it should have been expected. Thus in the case of grating no. 3 there was only one eighth of the energy in the condition for first order interference at $\lambda = 5.6\mu$ that there was in the condition for second order interference at $\lambda = 2.8\mu$. Also the dispersion of the rock-salt prism is greater in the region around $\lambda = 5\mu$ than it is at the shorter wave-length and this would tend to widen the first order interference band and make it less marked.

Grating no. 4 differed from the others in that there was a much larger portion of the original surface of the speculum plate left undisturbed. The behavior of this grating in the infra-red spectrum was unsatisfactory from the standpoint of concentration of energy into a single order. The most satisfactory grating from this standpoint was no. 5 and for this reason this grating will be described somewhat in detail.

Grating No. 5.

Area of ruled surface 2 square inches. Grating constant 0.0011508 cm. A collodion cast of the grating was made and a piece was embedded in

[1] Although it was not noted at the time the observations of Trowbridge and Crandall on which the paper in the Phil. Mag. already cited was based show that the gratings nos. 6 and 7 made on gilded plates exhibit the same interference phenomenon in the central image though it was far less marked than in the present case.

In the case of the grating no. 7 ruled by Professor Wood the form and angle of the groove were very accurately known and the retardation over the element when the central image was on the bolometer strip should have been $\rho_1 = 8.2\mu$ (see Fig. 5, p. 545, Phil. Mag.). There was a minimum in the curve of energy distribution which amounted to about 20 per cent. at wave-length $\lambda = 2.8\mu$ $\therefore \rho_0/\lambda = 2.93$ waves or within the probable error of these observations the minimum was due to interference taking place because of a retardation over the element of three waves.

There was no trace of minima due to one and two waves retardation.

paraffine and cut in thin sections by a microtome. The sections were cut so as to give a profile view of the cut made by the diamond point of the ruling engine, *i. e.*, the view one would have if one were to look along the plane of the grating parallel to the direction of the grooves. The microtome section was mounted in the usual way for microscopic examination and a fair estimate of the general character of the groove was obtained. The collodion section showed that from 80 per cent. to 90 per cent. of the original surface had been removed and that the cut was very nearly triangular in cross-section with the corners slightly rounded off. It was thought that this was in part due to capillary action having prevented the collodion from completely filling the mould and that the cut was probably more nearly triangular than the section indicated it to be. One side of the cut was very nearly at right angles to the original surface and the other, and longer, side made an angle of from ten to eleven degrees with the original surface. There was no noticeable difference in the character of the cut at the two ends of the grating such as one might expect to find if the diamond point had worn down or if the density of the speculum metal had not been the same at all points Considering that the cut was a triangle about 1.5μ deep and 9μ wide there was a surprising uniformity in its character all across the surface of the grating.

The curves of Plate 3 give the energy distribution for different settings of this grating when the energy incident on the grating was of the mean wave-length marked under the curve. The abscissæ represent the angles through which the grating was turned away from the position in which the central image was on the bolometer strip. The ordinates of the curves represent the galvanometer deflections and are proportional to the energy falling on the bolometer strip in the given position.

The order of the spectrum is marked near the maximum of any curve and the deflection observed when the central image was on the bolometer strip is marked C.I. in each case. The central image was too narrow to be represented on the scale to which the remainder of the figure is drawn. The position of the "oblique image" is marked o.i. on the plates. To save space in reproducing the plates only the region near the strong order or orders is represented.

The curves show that in general there is a marked concentration of energy into the order or orders near the oblique image. In the case of the first of the curves (that for $\lambda = 1.1\mu$) the fourth order is nearer to the oblique image than the third order is and the fourth order contains the greater amount of energy. For $\lambda = 1.65\mu$ the second and third orders are about equidistant from the oblique image and they contain about equal amounts of energy.

With the exception of the case for $\lambda = 2.8\mu$ it is generally true that the order nearest to the oblique image contains the greatest amount of energy.

The second order maximum for $\lambda = 2.1\mu$ comes almost exactly in the position of the oblique image and in this case 90 per cent. of the incident energy is concentrated into this one order. In estimating energy the area under an energy maximum was taken to be proportional to the energy in the order in question and the ratio of this area to the total area under all the maxima including the central image was taken to measure the per cent. of the incident energy present in the order in question.

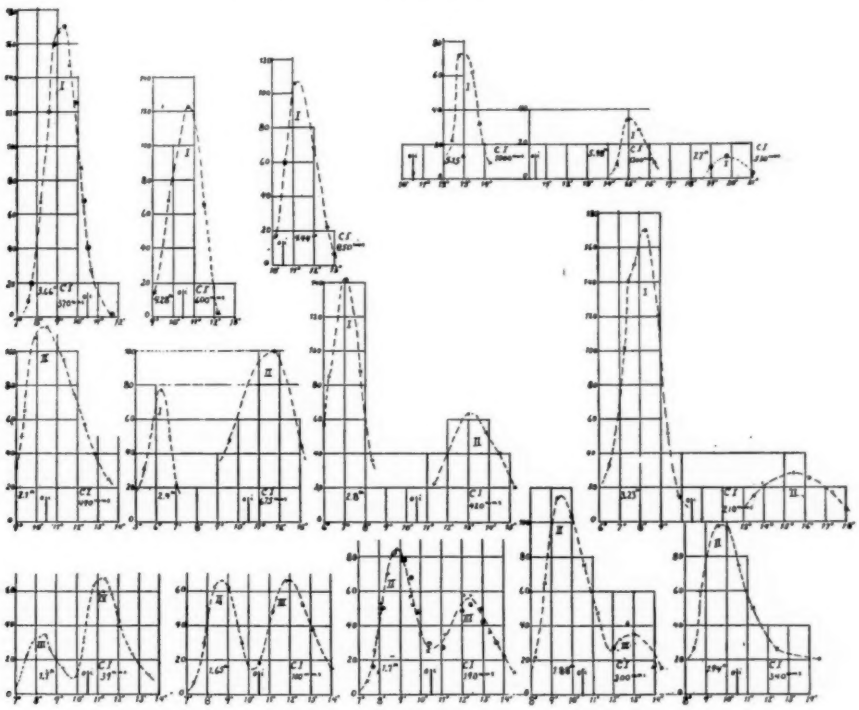


Fig. 3.

For $\lambda = 1.65\mu$ where the oblique image comes midway between the second and third orders there is present about 40 per cent. in each of the orders. For $\lambda = 2.8\mu$ about 50 per cent. only of the energy is present in the strong first order while for $\lambda = 3.66\mu$ for which the first order is formed close to the oblique image there is a concentration of 90 per cent. into this order. In the above one is strongly reminded of the "single and double order" of the echelon.

After the first order has passed beyond the position of the oblique

image there is a rapid fall off in concentration; thus at $\lambda = 5\mu$ 50 per cent.; at $\lambda = 6\mu$ 25 per cent., etc.

The curves show that this grating concentrated as much as 90 per cent. of the available energy into the order which is formed in the position of the oblique image and that it is particularly suitable for use in the infra-red spectrum in the neighborhood of $\lambda = 3\mu$ for in this region there is strong concentration into the first order combined with destructive interference in the central image. It should be possible to rule a number of gratings each one of which was as good as that just described in some other wave-length interval and such a set of gratings would prove a valuable adjunct in the spectroscopy of the infra-red region for they would combine a high resolving power with an energy concentration comparable with that in a prismatic spectrum.

To further test the validity of Eq. A (page 491) the behavior of grating no. 5 was studied with visible light. The radiation from a Cooper-Hewett quartz mercury vapor lamp was focused on the slit of a spectrometer on the table of which the grating was mounted. The objective was set on some one of the orders of the blue line ($\lambda = 0.4359\mu$) and the grating table and the objective were moved so as to always keep the given order in the field of view. In general one or more minima of intensity could be observed as the angle of incidence was varied and frequently these minima were so sharp as to amount practically to the disappearance of the order. The angles θ and δ of Eq. A were noted for each of these minima and this was done for each of the various orders of the blue line on the concentration side of the grating. The quantities a and b in Eq. A were approximately known from examination of the microtome section of the cast taken from the grating so with these values as a first approximation it was attempted to obtain values for a and b which should make the calculated retardation some whole number of waves for all the observed values of θ and δ for which the various orders were weak or vanished. The following table gives the results of these visual observations and the calculations.

The first column gives the order of the weak or absent line; the second gives the half angle between the collimators when the line was weakest or vanished; the third gives the deviation δ from the position of the central image and the last column gives the value of the retardation ρ in wave-lengths of the blue light calculated from Eq. A.

If the formula is correct and all observations were without error this last column should contain only whole numbers. Considering the difficulty of estimating intensity with the unaided eye the numbers in the last column do not differ from whole numbers by more than one would be inclined to judge the probable error to be.

TABLE.

Weak or Absent Orders.	θ	δ	ρ/λ	Weak or Absent Orders.	θ	δ	$\rho\,\lambda$
1	33° 45′	9° 17′	4.99	3	40° 09′	6° 19′	2.99
1	42° 24′	9° 07′	4.17	3	50° 49′	5° 25′	1.97
1	51° 39′	8° 50′	3.20	3	59° 31′	4° 09′	1.08
2	25° 14′	8° 11′	4.93	4	32° 34′	5° 25′	2.93
2	36° 14′	7° 53′	4.04	5	21° 54′	4° 43′	2.92
2	46° 19′	7° 26′	3.09	5	33° 14′	4° 05′	2.16
2	55° 24′	6° 46′	2.14	11	34° 30′	4° 04′	2.02
3	28° 49′	6° 52′	3.94	12	37° 24′	6° 03′	2.81

From the dimensions of the groove given at the beginning of this table it may be calculated that the retardation over the element when the central image was on the bolometer strip in the previous experiment with infra-red energy should have been $\rho_0 = 3.09\mu$. As previously stated it was found experimentally to be 3.2μ.

With visible light those orders were most intense which lay nearest the position of the oblique image as was the case when the grating was examined with infra-red energy. The experiment with visible light shows that the perfection of the ruling was considerably greater than that previously obtained with the echelette gratings ruled on gilded plates. The depth and width of the cut was evidently more constant over the ruled surface as there was not the slightest change in the character of the illumination in the orders when a small diaphragm was moved around over the surface of the grating.

GRATING NO. 3.

Area of ruled surface 2.75 sq. inches.

Grating constant $g = 0.0011508$ cm.

The section of a collodion cast taken from this grating showed that practically none of the original surface of the speculum flat was left and that the angles of the new planes were about 15° and 90°.

The graphs of the energy distribution among the various orders when energy of a certain wave-length interval was incident were very similar to those shown for the case of grating no. 5 except that the oblique image came about 15° away from the central image and that for the shorter wave-lengths of the infra-red spectrum certain of the lower orders were weak or absent.

When energy of mean wave-length $\lambda = 0.99\mu$ was incident the second, third and fourth orders on the concentration side were very weak, the first, fifth and seventh were weak and the concentration came in the sixth order which was formed near the position of the oblique image.

Using Eq. A with the values $b = 11.51\mu$ and $a = 15° 10'$ the retardation in waves for the absent orders are 3.9 for the second, 2.9 for the third, and 1.9 for the fourth or approximately a whole number of waves in each case. When the incident energy was of mean wave-length 1.18μ the fifth order was very strong and the second was absent—for the latter the retardation over the element was 2.9 waves.

With incident energy of mean wave-length 1.51μ the third and fourth orders were strong and the second order was absent—for it the retardation was 2.9 waves. In general the concentration of energy into the order near the oblique image already noted in the case of grating no. 5 is also evident in the graphs for grating no. 3.

This grating was not so satisfactory with visible light as was grating no. 5 though it showed the phenomenon of weak or absent orders in rough accord with calculation from the dimensions of the groove. The edges of the cut were evidently badly chipped so the performance of the grating could not be expected to be so good with light as with the infra-red.

GRATING NO. 4.

This grating was ruled with the same point as grating no. 3 but the depth of the cut was not sufficient for the grating to show marked concentration of energy into the direction of the oblique image. However the perfection of the ruling was much greater than it was in the case of no. 3 and the weak orders with visible light came about as predicted by Eq. A.

TABLE.

Weak or Absent Orders.	θ	δ	ρ/λ
12th yellow Hg.	31° 38′	20° 46′	1.00
11th " "	39 45	21 11	1.01
6th " "	39 23	11 16	1.00
4th " "	52 18	9 28	0.95
2d " "	48 40	4 22	2.04
16th blue "	25 38	19 38	1.13
4th " "	38 35	5 34	3.05
13th green "	27 30	20 21	1.05
2d " "	49 30	4 11	2.10

The numbers in the last column are close enough to integers to explain the absence of the orders if the inclined element alone be considered. However in the case of this grating where only about one half of the original speculum surface is recut into inclined elements one should expect all of the even orders to be absent anyway even if the inclined elements contributed nothing to the spectrum by diffraction. It will

be noted that nearly all of the absent orders in the table are even orders so there remain only two observations which confirm Eq. A.

While the technique of ruling a polished triangular cut in a hard brittle alloy like speculum metal is by no means perfected at least one of the gratings described (no. 5) satisfies very closely the elementary theory of diffraction from a triangular element even for light of short wave-length while the observations with the other gratings are in good agreement when energy of longer wave-length is used.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY, June, 1916.

THE FREQUENT BURSTING OF HOT WATER PIPES IN HOUSEHOLD PLUMBING SYSTEMS.

BY F. C. BROWN.

IT seems to have been a common observation by plumbers that the pipes carrying the hot water from the furnace to the kitchen and bath room burst from freezing more frequently than do the pipes carrying the cold water. It is said that the ratio of frequency is at least four to one. The "cold water" usually freezes so as to lessen the flow of water in the pipes or to stop it entirely, but this freezing is seldom accompanied by bursting. If a long connection of pipe falls below the freezing temperature the probability of bursting is enhanced by virtue of its length. The bursting of the cold-water pipes is generally expected when the freezing temperature is very low. That "hot water" pipes should burst more readily than those carrying cold water might appear anomalous. Thus the experiments described in this paper were undertaken first to verify the observations of the plumbers and then to ascertain the physical explanation.

In the experiments an exact duplication of the conditions in the plumbing system were not obtained, but they were approached in some essentials by using glass test tubes closed at one end instead of iron pipe. Ordinary tap water freshly drawn was placed in these tubes to simulate the conditions in the cold-water pipes. The same tap water shortly after being boiled was placed in like tubes to simulate the conditions in the hot-water pipes. The glass tubes were satisfactory in that the visible appearances inside the tube offered valuable information as to what was happening in the respective cases. The breaking stress of the glass was materially lower than that of the ordinary iron pipe, but expected differences from this cause would merely increase the number of tubes bursting in a given time. The ordinary iron pipe that is stopped by freezing may generally be regarded as an open pipe, contrary to the glass tubes used which had one end closed. The closed pipe would have an increased probability of bursting, because of the lessened opportunity for the release of pressure by the escape of water and ice.

The glass tubes seemed to satisfactorily duplicate the conditions of the plumbing system in that the tubes containing the boiled water did

burst more frequently than those with the unboiled water. In the preliminary trials, test tubes of varying size and number were placed in the open air when the outdoor temperatures were at varying degrees below zero. The time for freezing varied from thirty minutes to four hours. The hot and cold water were placed in alternate tubes side by side. The tubes were usually about one centimeter in diameter. On seven occasions about 50 pairs of tubes were tried. In every test the tubes of boiled water broke first. When the observations were concluded it was noted that 44 tubes containing boiled water had burst and only four of those filled with unboiled water had broken. For one reason or another slightly more than half of the tubes did not break, even though all the water seemed to be frozen. Having verified the observations of the plumbers with the altered conditions, the next step was to obtain the explanation. The rate of cooling and the temperature at which freezing occurred furnished information that limited the possible explanations. In this experiment both kinds of water were allowed to reach approximate room temperatures before placement in the freezing surroundings outside.

It was found that the boiled water was invariably undercooled several degrees below zero before freezing commenced and that after crystallization began the temperature remained at zero until the entire mass was frozen. The ice was quite clear and solid. The unboiled water always began freezing at zero and the ice was full of air bubbles and appeared quite slushy, particularly near the central axis of the tube. These observations are quite in accordance with the well-known differences existing between the freezing of air-free water and water saturated with air. H. T. Barnes[1] states that the more the air and ice are mixed the less can the mixture be cooled below zero. When chilled air was passed through the water he obtained frazil or slush ice, lacking in solid structure. He further states that "with agitation, and the presence of dust or suspended matter, and particularly with dissolved air in the water, super-cooling without ice forming may be said to be impossible." Thus the observed differences between the freezing of boiled and unboiled tap water are such as can be explained by the presence or absence of dissolved air. The white spongy ice core along the central axis of the tubes in our experiments approaches in a small way the properties of frazil ice.

The fact that the temperature of the tap water during freezing did not fall below zero is indicative that the impurities, such as dissolved salts, were comparatively small. The explanation then seemed rather

[1] Ice Formation, published by Wiley, 1906.

to involve the super-cooling of the boiled water and the more solid ice formed in it. Both of these are largely dependent on the diminished amount of air in the boiled water.

The ice formed from the boiled water was clear without a decided central region often observed in artificial ice, while the ice formed from tap water was full of air bubbles especially toward the middle. These air bubbles together with accompanying impurities form nuclei for crystallization and also act as cushions to react against the expansion of the water by freezing. The air also tends to freeze out toward the middle. It will be shown that this semi-open center acts as a safety valve to relieve the pressure caused by freezing. The undercooling of the air-free water tends to make the center of the tube freeze just as solid as the water near the surface of the tube.

The occluded air also seems to make the ice more fluid in its nature. In 28 tubes side by side, which were alternately filled with boiled and unboiled water, to the same depth in each case, there was observed in every case to be a greater rise of ice in the tubes in which the unboiled water froze. Quantitative heat measurements showed that approximately the same amount of ice was formed in each case. Therefore the ice from the tap water either was less viscous than the other ice or the central core of the tap water ice remained mobile sufficiently long to relieve the pressure by carrying water and ice crystals along the tube to regions of less stress. Koch[1] found the ice containing air bubbles to have a lower elasticity than air free ice. In other words air bubbles weakens the ice. Quincke[2] observed when water containing both air and dissolved salts began to freeze that both the air and salts separated out from the ice alike, leaving the mother liquid excessively rich in both air and salts. This explains why the central core in the frozen tap water was particularly full of air bubbles. This central core either remained open or froze into a slushy ice mixture. In either case this formed an easy avenue for the release of the water pressure that tended to exist by virtue of the ice freezing along the outer wall of the tube.

The foregoing conclusion that the air in ordinary tap water is responsible for the non-bursting of cold-water pipes in many instances was further checked as follows. Water that had been boiled was afterward saturated with air by passing a stream of air through it for several minutes. Under these circumstances six pairs of tubes, alternately boiled and unboiled water, burst at about the same time. However the bursting was delayed as compared with air-free water even when the freezing temperature was quite low.

[1] Ann. d. Phys., 41, pp. 709–727, 1913.

[2] Proc. Roy. Soc. Canada, 3, p. 24, 1909.

A further experiment was made to be certain that the presence of dissolved salts was of no special importance to the observed phenomenon. Distilled water which had been exposed to the air for several days was substituted for the tap water. One of every pair of tubes was filled with this unboiled distilled water. Every alternate tube was filled with the same water after it had been reboiled. As expected the tubes containing the boiled water burst first. A final test was made by saturating this distilled boiled water with air. This procedure caused all the tubes to behave alike in freezing surroundings.

In view of these experiments it has been concluded that the occluded air in ordinary tap water is responsible for the delay or absence of bursting of the pipes. The air and accompanying impurities assist in furnishing nuclei of crystallization, so that the ordinary tap water begins to freeze at zero degree. At the same time the ice formed is more mobile, especially near the middle of the tube, so that until very low temperatures are reached the pressure is released along the middle of the tube by the flow of water and ice. In addition the air bubbles displacing water form cushions which relieve the pressure on the tube to a certain extent.

The writer is indebted to Mr. Waldemar Noll for very able assistance, particularly in connection with the experimental work here described.

ON THE DEMAGNETIZATION OF IRON AND STEEL RODS BY STRAIN AND IMPACT.

BY GUY G. BECKNELL.

THE following is an investigation concerning the redistribution of magnetic flux through the various cross-sections of magnetized rods, after they have been subjected to torsion, flexure, or to the impact of a hammer.

The rods used had a diameter of about 1.25 cm., and were 112 cm. in length. They were first completely demagnetized by means of a gradually diminishing alternating current passing through the turns of a helix of about the same length as the rod. Then by means of a direct current through the same helix, the rods were magnetized to saturation, or nearly so.

In order to determine the number of lines of magnetic force passing through any cross-section of the magnet, a narrow spool of small diameter, wound with fine copper wire, was employed, the terminals being connected to a sensitive ballistic galvanometer. The spool, being placed on the rod at the section desired, was slid off from that point entirely over the end of the rod, thus cutting all the lines of force which escaped from the magnet in the length passed over. If the spool is moved with sufficient rapidity, the throw produced in the ballistic galvanometer is proportional to the total magnetic flux at the cross-section specified.

The dimensions of the spool were as follows: diameter 4 cm., width of windings 1 cm., and the wire was chosen of such a diameter that its total resistance was equal to that of the galvanometer, the condition for a maximum induced current.

In brief, then, the method was to determine the induction flux at various sections along the magnetized rod, after which the specimen was subjected to some form of mechanical strain and then again explored by the sliding coil; the remaining induction flux being measured at the same points on the rod as before. The decrease in this magnetic flux at any cross-section, occasioned by the mechanical strain to which the magnet was subjected, is here defined as the *demagnetization* at that section.

If we represent by x the linear coördinate specifying the particular

section of the rod investigated, by ϕ_x the flux at that section before strain, and by ϕ_x' the flux there after strain, we have

$$D_x = \phi_x - \phi_x', \tag{1}$$

where D_x is the demagnetization over the section x.

TORSION.

Perhaps the simplest case to investigate in this connection is that of uniform torsion of the magnet.

The specimen was held securely at one end in a horizontal position by means of a heavy brass clamp which gripped about one centimeter of the rod. Emery dust sprinkled over the rounded jaws of the brass vise was sufficient to prevent all slipping.

Near the free end of the magnet another heavy brass clamp was placed to prevent flexure in any direction, the jaws being left loose enough to allow the rod to rotate easily during torsion. At the extreme free end was a strong brass lever, fastened at right angles to the specimen by means of a brass key-pin passing through a small hole in the iron. During torsion, especially for large strains, it was found necessary to use a loose-fitting clamp at the middle to do away with buckling of the specimen which had caused large errors in the results. The angle of torsion was measured with sufficient accuracy by using a light metal pointer of length 30 cm., fastened at right angles to the rod, and moving over a scale.

There was no difficulty found in demagnetizing a specimen, and re-magnetizing it to a standard distribution, but it did require practice to be able to subject the specimen to a given torsion and leave it in the same state of demagnetization time after time. Especially was this true of Norway iron, which may be appreciably demagnetized merely by stroking. This is due perhaps to vibrations set up within the metal, and not merely to surface agitation.

The rods were investigated when in the direction of magnetic east and west, but this was found to be a hardly necessary precaution.

In Tables I. and II. are shown the results of uniform torsion on a magnet of Norway iron. The clamp was applied from $x = 0$, to $x = 1$ cm., and the torsion at $x = 112$ cm.

The angle of torsion here merely represents the angular displacement of the free end of the specimen with respect to the fixed one. Results are given for six different angles, for the highest of which the rod showed an appreciable set after being released from torsion.

It should be stated at this point that the rods were studied not while under strain, but after release.

The first column in the tables specifies the section of the magnet, the second gives the original magnetic flux at that section. In the first

TABLE I.

NORWAY IRON.

Demagnetization by Uniform Torsion.

x Cm.	ϕ_x.	Torsion 1.5°. Factor 0.216.		Torsion 2.4°. Factor 0.320.		Torsion 3.6°. Factor 0.436.	
		Observed D_x.	Computed D_x.	Obs. D_x.	Comp. D_x.	Obs. D_x.	Comp. D_x.
2	145	28	31	37	46	63	63
5	315	68	68	97	101	139	137
8	464	102	100	150	149	206	202
11	599	133	129	198	192	265	261
14	723	160	156	241	231	319	315
17	837	187	181	281	268	368	365
23	1,037	226	224	351	332	453	452
29	1,200	261	259	397	384	524	523
35	1,331	288	288	451	426	580	581
44	1,465	317	316	494	469	634	639
56	1,532	331	331	513	490	661	668
68	1,474	318	318	492	472	635	643
77	1,345	290	290	448	430	580	586
83	1,215	263	262	405	389	525	530
89	1,043	228	225	349	334	455	455
95	850	186	184	281	272	369	371
98	739	162	160	241	237	320	322
101	615	136	133	199	197	265	268
104	490	106	106	150	157	206	214
107	329	73	71	98	105	140	143
110	161	36	35	37	52	67	70

of each of the double columns following is the corresponding demagnetization as measured for the torsional angle specified.

It will be found that in each case the demagnetization, D_x, divided by the original flux, ϕ_x, gives nearly a constant ratio, R, for a fixed angle of torsion.

The mean of these ratios was taken over the entire rod for each torsion angle, θ, and the values of R and θ were found to be quite accurately related by the equation

$$aR = \theta(b - R^2) \qquad \text{(see Fig. 1)}, \tag{2}$$

where a and b are constants depending upon the material. For the specimen of Norway iron used, the values of these constants were found to be $a = 7.585$; $b = 1.115$.

In Tables I. and II., following, the column showing the values for the

TABLE II.

NORWAY IRON.

Demagnetization by Uniform Torsion.

x Cm.	ϕ_x.	Torsion 5.2°. Factor 0.556.		Torsion 8.1°. Factor 0.687.		Torsion 11.3°. Factor 0.773.	
		Obs. D_x	Comp. D_x.	Obs. D_x.	Comp. D_x.	Obs. D_x.	Comp. D_x.
2	145	85	81	86	100	101	112
5	315	184	175	204	216	235	243
8	464	251	258	307	319	352	359
11	599	329	333	402	412	458	463
14	723	400	402	486	497	554	559
17	837	464	465	564	575	642	647
23	1,037	574	577	700	713	797	802
29	1,200	666	667	813	825	927	928
35	1,331	740	740	904	914	1,029	1,029
44	1,465	816	814	999	1,006	1,136	1,132
56	1,532	854	852	1,045	1,052	1,190	1,184
68	1,474	820	819	1,006	1,013	1,142	1,139
77	1,345	747	748	922	924	1,041	1,039
83	1,215	676	676	838	835	943	939
89	1,043	587	580	730	717	809	806
95	850	478	473	598	584	668	657
98	739	415	411	522	508	581	571
101	615	346	342	438	423	485	475
104	490	269	272	344	337	381	378
107	329	183	183	240	226	266	255
110	161	85	90	120	111	135	124

demagnetization as measured for a particular torsion angle, is a column in which the computed value $D = R\varphi$ of the demagnetization is shown. The factor R, specified at the top of the column, is computed from equation (2) for each torsion angle.

It will be seen that the agreement between the observed and calculated values of D is good in general, the variation being not often greater than 1 or 2 per cent. of the maximum demagnetization measured. Of course the greatest deviations are at the ends where the quantity measured is least. Also the magnetic distribution at the ends is somewhat changed by the clamped portion and by the torsion device.

From this it may be seen that if we have given the original flux curve and the two constants a and b, the demagnetization curve for any angle θ (within the elastic limit) is determined by the two equations

$$aR = \theta(b - R^2) \tag{3}$$

$$D_x = R\phi_x,$$

or by

$$D_x = \frac{\phi_x}{2\theta}\left[\sqrt{4b\theta^2 + a^2} - a\right].$$

A similar set of experiments was performed also with a specimen of Viking steel, and the results are here tabulated (Table III.).

TABLE III.

VIKING STEEL.

Demagnetization by Uniform Torsion.

x Cm.	ϕ_x.	Torsion 4.1°. Factor 0.128.		Torsion 8.1°. Factor 0.234.		Torsion 12.6°. Factor 0.328.		Torsion 16.5°. Factor 0.390.		Torsion 20.6°. Factor 0.440.	
		Obs. D_x.	Comp. D_x.	Obs. D_x.	Comp. D_x.	Obs. D_x.	Comp. D_x.	Obs. D_x.	Comp. D_x.	Obs. D_x.	Comp. D_x.
2	628	73	80	95	147	135	206	262	245	313	276
5	1,319	175	169	283	309	394	433	555	514	641	580
8	1,912	259	245	440	448	612	627	800	746	919	841
11	2,437	330	312	579	570	803	799	1,012	950	1,161	1,072
14	2,903	390	372	700	679	968	952	1,196	1,132	1,371	1,277
17	3,321	442	423	804	777	1,113	1,089	1,358	1,295	1,555	1,461
23	4,032	526	515	975	943	1,349	1,319	1,620	1,568	1,844	1,774
29	4,571	589	585	1,100	1,070	1,524	1,499	1,814	1,783	2,049	2,011
35	4,991	636	639	1,186	1,168	1,649	1,637	1,951	1,947	2,189	2,196
44	5,403	679	692	1,261	1,264	1,757	1,772	2,070	2,107	2,316	2,377
56	5,605	698	717	1,295	1,312	1,802	1,838	2,122	2,186	2,374	2,466
68	5,400	675	691	1,263	1,264	1,761	1,771	2,071	2,106	2,318	2,376
77	4,994	626	639	1,190	1,169	1,663	1,638	1,954	1,948	2,192	2,197
83	4,585	575	587	1,108	1,073	1,547	1,504	1,819	1,788	2,052	2,017
89	4,044	510	518	992	946	1,376	1,326	1,626	1,577	1,848	1,779
95	3,352	424	429	835	784	1,147	1,100	1,365	1,307	1,567	1,475
98	2,938	374	376	739	688	1,011	964	1,205	1,146	1,394	1,293
101	2,471	318	316	629	578	852	811	1,023	964	1,193	1,087
104	1,947	254	249	502	456	670	639	811	769	967	857
107	1,354	183	173	353	317	461	444	568	528	706	596
110	671	99	86	177	157	215	220	281	262	395	295

The relation between R and θ was found to be of the same form as that for Norway iron (equation 2), the values of the constants a and b being respectively 18.241 and 0.583.

The agreement between the observed and the calculated values of the demagnetization is not so good as with Norway iron, the discrepancies in a few instances being as great as 4 or 5 per cent. of the maximum demagnetization measured. However, the angles of torsion ran higher, and on account of the necessarily greater torsional moment required the specimen showed the effect of buckling between the clamp-guides to a greater extent than did the first specimen.

The curves representing R as a function of θ for the two materials are shown in Fig. 1.

It is evident at once that the soft iron is far more subject to torsional demagnetization than the steel. For an angle of 1.75° the Norway iron had lost at each cross-section, one fourth of its original flux at that section; for a torsion of 4.4°, one half; and for 10.3°, three fourths. While the Viking steel had lost only one fourth at 8.8°, and as far as 20° had not lost half the original flux.

In Fig. 2 are shown the demagnetization curves for Viking steel; *G*, *F*, *E*, *D* and *C* corresponding to angles of torsion of 4.1°, 8.1°, 12.6°,

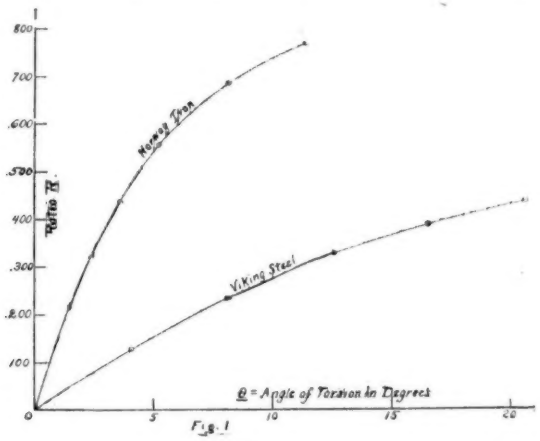


Fig. 1.

16.5° and 20.6° respectively. Since these angles vary by increments of about 4°, it may be seen, since the curves become nearer together for the greater angles, that the demagnetization is relatively greater for the small angles of torsion.

The curve *A* represents the flux before torsion as a function of *x*. *B* is the flux-curve after torsion, for an angle of 16.5°. Curve *D*, then, is the difference of the curves *A* and *B*.

A is approximately a parabola, and is represented by the equation

$$Ax^2 + Bx + C = \phi_x, \tag{4}$$

where *A*, *B* and *C* are constants. Since it was shown that $D_x = R\phi_x$, where *R* is a constant for a fixed angle of torsion, it follows that

$$D_x = A'x^2 + B'x + C', \tag{5}$$

where $A' = AR$; $B' = BR$; $C' = CR$, and so the demagnetization curves are also approximate parabolas.

The following table (IV) shows the agreement between the observed data and the parabolic equation for the original flux curve with Norway iron.

By the method of least-squares the constants best fitting the observations were found to be $A = -0.4737$; $B = +53.18$; $C = +54.65$.

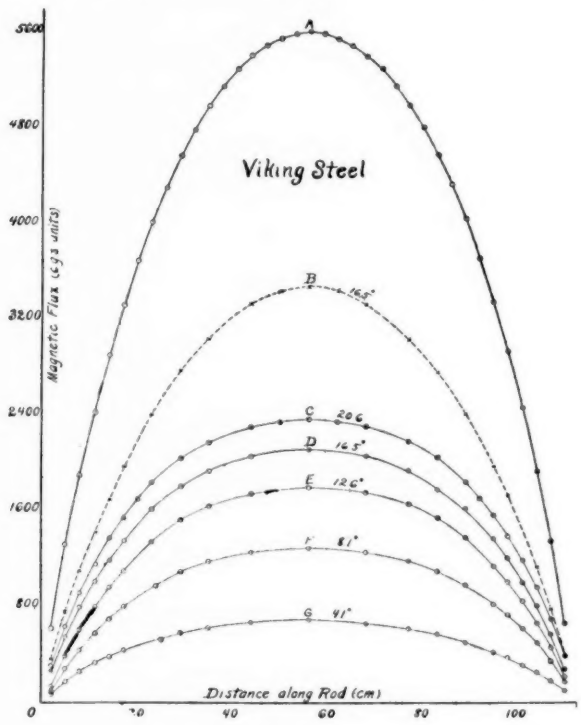


Fig. 2.

In Table IV. are also shown three sets of data for the demagnetization curves, the constants of the parabolic equation being found in each case by multiplying the values of A, B and C above by the factor R corresponding to the particular torsion angle. In other words the columns of computed values of demagnetization are derived for the torsion angles 1.5°, 3.6° and 5.2°, by multiplying the corresponding values of ϕ by the factors 0.216, 0.436, 0.556 respectively, as derived from Equation 2.

Thus the entire family of demagnetization curves corresponding to values of θ within the elastic limit may, by knowing the values of five constants, A, B, C, a, b, be constructed from the two equations

$$aR = \theta(b - R^2), \tag{6}$$

$$D = ARx^2 + BRx + CR.$$

In Table V. is shown the behavior of the specimen of Viking steel when subjected to a fixed torsion, 8.1°, but starting with different intensities of magnetization. In the first case the magnetizing current was 0.2

TABLE IV.

Norway Iron.

x Cm.	ϕ_x Obs.	ϕ_x Comp.	1.5°.		.6°.		5.2°.	
			D_x Obs.	D_x Comp.	D_x Obs.	D_x Comp.	D_x Obs.	D_x Comp.
2	145	159	28	34	63	69	85	89
5	315	307	68	66	139	134	184	171
11	599	582	133	126	265	254	329	324
14	723	706	160	153	319	308	400	393
23	1,037	1,027	226	222	453	448	574	571
29	1,200	1,199	261	259	524	523	666	666
35	1,331	1,336	288	289	580	582	740	743
44	1,465	1,478	317	319	634	644	816	822
56	1,532	1,547	331	334	661	675	854	871
68	1,474	1,481	318	320	635	646	820	823
77	1,345	1,341	290	290	580	585	747	746
83	1,215	1,205	263	260	525	526	676	670
89	1,043	1,036	228	224	455	452	587	576
98	739	717	162	155	320	313	415	399
101	615	594	136	128	265	259	346	330
107	329	320	73	69	140	140	183	178
110	161	171	36	37	67	75	85	95

TABLE V.

Viking Steel.

Torsion Angle = 8.1°.

x Cm.	ϕ.	$R = 0.272$.	ϕ.	$R = 0.245$.	ϕ.	$R = 0.234$.
		D.		D.		D.
2	266	50	579	92	628	95
5	487	115	1,141	259	1,319	283
8	694	179	1,642	410	1,912	440
11	890	241	2,100	545	2,437	579
14	1,075	298	2,520	664	2,903	700
17	1,248	351	2,903	769	3,321	804
23	1,560	440	3,721	941	4,032	975
29	1,821	511	4,121	1,066	4,571	1,100
35	2,031	564	4,542	1,152	4,991	1,186
44	2,255	615	4,957	1,223	5,403	1,261
56	2,366	637	5,155	1,247	5,605	1,295
68	2,270	613	4,957	1,197	5,400	1,263
77	2,065	571	4,542	1,085	4,994	1,190
83	1,863	525	4,124	1,035	4,585	1,108
89	1,607	460	3,583	913	4,044	992
95	1,303	377	2,920	750	3,352	835
98	1,131	328	2,540	653	2.938	739
101	946	275	2,125	544	2,471	629
104	747	216	1,670	422	1,947	502
107	533	154	1,163	285	1,354	353
110	304	90	608	130	671	177

ampère; in the second 0.5 ampère, and in the third, 4.0 ampères, sufficient to produce approximate saturation.

The flux curves as well as the demagnetization curves are all approximate parabolas as before, and so there is nearly a constant ratio between the demagnetization at any point and the original flux corresponding. The mean ratios were found to be respectively 0.272, 0.245, 0.234, showing that the lower the original intensity of magnetization, the greater the *relative* demagnetization. However, the actual demagnetization at every section of the magnet is seen to be greater for higher intensities.

If these values of R are considered as functions of the original flux at the middle section, the relation is found to be linear, but the data at hand are not sufficient to consider this as accurately true over a wide range of values of initial flux.

A much more complicated problem arises when the magnetized rod is clamped at some point other than the end, and torsion applied at one of the free ends. In this case the portion behind the clamp is not subjected to strain at all, while that in front of the clamp undergoes uniform torsion. The demagnetization curve, of course, is continuous as before, but it is no longer symmetrical.

In Table VI. are shown three sets of data representing the demagnetization corresponding to each of three different torsion angles, when the specimen is clamped at the middle point ($x = 56$ cm.) and twisted at the free end, $x = 112$ cm.

The second column of the table gives the values of the initial flux for the various values of x. There does not seem to be any simple relation between D_x and ϕ_x in this case, although it was found that the values of D at $x = 56$ are accurately proportional to the ratio of the corresponding angles of torsion. This ratio is nearly the same for all values of x, and the lack of uniformity is perhaps due to slight flexure during torsion.

One point of interest may be noticed, that the maximum demagnetization for all angles of torsion occurs at $x = 77$, which is three eighths of the distance from the clamp to the twisted end of the magnet. It will be seen later that in several other instances under torsion and fluxure the maxima occur at multiples of 7. (7 is one sixteenth the length of the rod.)

A similar set of data is shown in Table VII. The specimen in this case was clamped at the quarter-length ($x = 28$ cm.), and torsion applied at the free end, $x = 112$ cm. As before the values of the demagnetization at the mid-section, $x = 56$, are accurately proportional to the angles of torsion; and, in fact, the proportionality is fairly good throughout the length of the rod.

TABLE VI.

Torsion (Viking Steel).

Rod clamped at middle.

x Cm.	C.G.S.	1.58°.	3.16°.	4.74°.
	ϕ.	*D.*	*D.*	*D.*
2	720	81	82	167
8	2,036	112	150	282
14	3,036	136	208	381
20	3,838	158	262	467
26	4,452	178	315	547
32	4,936	198	370	628
38	5,291	238	427	693
44	5,541	249	491	795
50	5,688	287	567	896
56	5,731	337	666	1,019
62	5,679	370	746	1,129
68	5,533	385	791	1,196
74	5,279	391	815	1,230
77	5,118	392	817	1,236
80	4,928	389	812	1,227
86	4,447	370	775	1,172
92	3,828	338	703	1,069
98	3,041	292	595	907
104	2,059	232	446	688
110	778	147	245	386

TABLE VII.

Torsion (Viking Steel).

Rod clamped at quarter-length. (x = 28 cm.)

x Cm.	ϕ.	4.33°.	6.08°.	7.70°.
		D.	*D.*	*D.*
2	752	113	120	185
8	2,048	219	279	382
14	3,030	320	426	562
20	3,813	423	572	740
26	4,432	535	725	927
32	4,913	641	876	1,112
38	5,265	705	976	1,244
44	5,509	748	1,045	1,334
50	5,659	779	1,095	1,397
56	5,714	798	1,125	1,439
63	5,645	811	1,138	1,458
68	5,521	802	1,130	1,450
74	5,276	789	1,112	1,419
80	4,922	760	1,068	1,363
86	4,444	709	981	1,266
92	3,825	632	893	1,124
98	3,047	530	722	936
104	2,062	423	606	684
110	769	230	247	335

The maximum demagnetization for each of the angles occurs at $x = 63$, which is five twelfths of the distance from the clamp to the twisted end of the rod. Or, in other terms, the maxima in the two cases respectively occur at eleven and nine sixteenths the length of the rod.

In Fig. 3 are shown the initial flux curve, the flux curve after torsion, and their difference, the demagnetization curve, for the case of the magnet clamped at the middle and twisted at the end, $x = 112$.

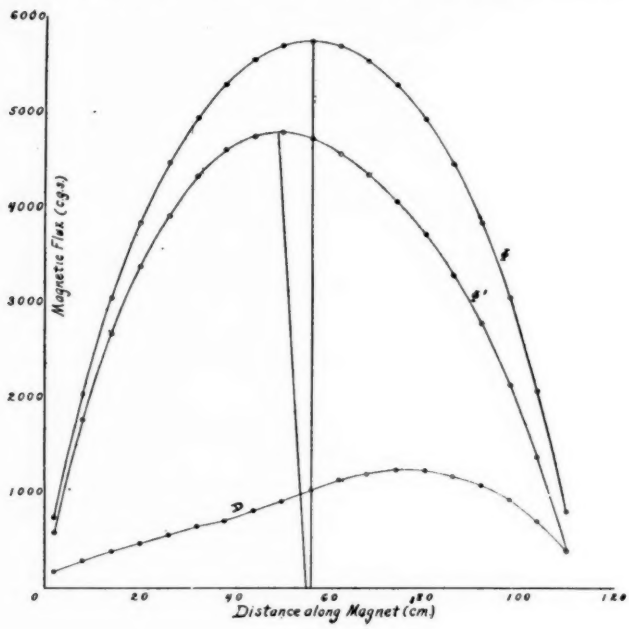


Fig. 3.

The first of these curves was found, as before, to be an approximate parabola of the form $\phi_x = Ax^2 + Bx + C$. The constants were found by the method of least squares to be: $A = -1.669$; $B = 187.8$; $C = 566.0$.

The second flux curve also resembles a parabola, but one which has had its axes rotated with respect to the first. Upon trial it is seen that this second curve is at least as nearly parabolic as the first, if not more accurately so. The equation of the parabola in this case is

$$(x + D\phi')^2 + Ex + F\phi' + G = 0,$$

where D, the tangent of the angle of rotation of the axes has a value of .00147 when ϕ' and x are plotted to the same scale. The other values of the constants found by least-squares are:

$$E = -113.66; \quad F = 0.51431; \quad G = -66.2.$$

In Table VIII. the observed and the computed values are given for the ordinates of these three curves. The demagnetization values agree

TABLE VIII.

Torsion (Viking Steel).

Rod clamped middle, torsion at end. Torsion angle 4.73°.

x	ϕ.		ϕ'.		$D\phi - \phi' = D$.	
Cm.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.
2	720	935	553	555	167	380
8	2,036	1,961	1,754	1,681	282	280
14	3,036	2,858	2,655	2,605	381	253
20	3,828	3,654	3,361	3,342	467	312
26	4,452	4,321	3,905	3,915	547	406
32	4,936	4,867	4,308	4,340	628	527
38	5,291	5,292	4,599	4,624	693	668
44	5,541	5,598	4,746	4,787	795	811
50	5,688	5,783	4,792	4,836	896	947
56	5,731	5,849	4,712	4,780	1,019	1,069
62	5,679	5,794	4,550	4,627	1,129	1,167
68	5,533	5,619	4,337	4,385	1,196	1,234
74	5,279	5,327	4,049	4,051	1,230	1,276
77	5,118	5,131	3,882	3,858	1,236	1,273
80	4,928	4,908	3,701	3,642	1,227	1,266
86	4,447	4,373	3,275	3,135	1,172	1,238
92	3,828	3,718	2,759	2,698	1,069	1,020
98	3,041	2,941	2,134	1,984	907	957
104	2,059	2,045	1,371	1,304	688	741
110	778	970	392	558	386	412

fairly well over a large range of values of x, but there are a few discrepancies for the smaller values of x. This need not be considered as fatal to the proposition that the demagnetization curve is the difference of two parabolas, one of which has been rotated with respect to the other. For in the first place, the original flux curve was not an exact parabola, and so the difference of two approximately fitted curves might show considerable discrepancies with respect to the observed data. It might be possible to produce an accurately parabolic distribution of flux by choosing properly the ratio of the diameter to the length of the magnet, or by varying the length of the magnetizing helix. In such a case the second flux curve might be, also, an accurate parabola.

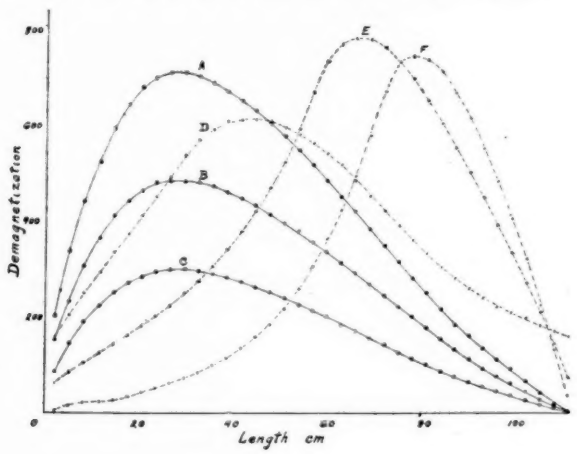
It may be worth while to mention that the greater the angle of torsion the greater the inclination of the axis of the second flux curve, but this inclination is relatively greater for small angles of torsion than for large values. Also, the inclination of the axis of the parabola is greater for the first method of clamping than for the second, when the strains are

equal in the two cases; but the two methods of applying torsion produce curves having the same general characteristics.

FLEXURE.

The first mode of flexure studied was with the magnet clamped at one end, the other end being slightly displaced. The curvature is then a linear function of the coördinate x, being proportional to the distance of the point from the free end of the rod.

In Table IX. the demagnetization is shown for displacements of the end of the rod of 2.3 cm., 3.6 cm., and 4.7 cm. In every instance there is a maximum in the value of D at about $x = 28$, or one fourth of the



(Flexure)

Fig. 4.

length of the rod. That is, the point of maximum demagnetization is midway between the point of maximum curvature and the point of maximum original flux. (See Fig. 4, A, B, and C.)

From this consideration it would seem reasonable that the demagnetization at any point is proportional both to the curvature and the original flux at the point. Or that the relative demagnetization, $R = D/\varphi$, is proportional to the curvature.

In column 5 of the table the values of R are shown for one of the experiments, and the variation is nearly linear toward zero at the free end.

Thus, we may write

$$D = k\phi(l - x), \tag{7}$$

where k is a constant, and l is the length of the rod under strain. However, if l is assumed to be a constant whose value, with that of k is to be

TABLE IX.

Tool Steel.

Rod clamped at end, $x = 0$; displaced at end, $x = 112$ cm.

x Cm.	ϕ.	Displacement at End, 2.3 Cm.			3.6 Cm.		4.7 Cm.	
		D Observed.	D Computed.	$R = D/\phi$.	D Observed.	D Computed.	D Observed.	D Computed.
2	1,420	88	80	.0620	140	141	203	190
5	2,425	149	133	.0612	235	220	339	316
8	3,278	192	174	.0587	308	289	445	415
11	4,021	227	208	.0564	367	344	529	495
14	4,680	253	234	.0540	412	389	599	559
17	5,259	273	255	.0519	445	423	648	609
20	5,766	287	271	.0498	468	450	682	646
23	6,212	296	282	.0477	483	468	702	673
26	6,601	300	290	.0455	489	483	711	691
29	6,938	300	294	.0433	489	488	711	701
32	7,229	297	295	.0411	484	490	704	704
35	7,477	292	294	.0377	475	487	692	700
38	7,684	285	290	.0371	464	481	676	690
41	7,851	276	284	.0351	450	471	656	677
44	7,894	266	277	.0333	436	459	633	660
47	8,081	254	268	.0315	419	444	607	638
50	8,148	242	257	.0297	399	427	580	613
53	8,182	230	246	.0281	380	408	551	586
56	8,191	216	233	.0264	360	387	520	556
59	8,168	202	220	.0247	338	365	488	525
62	8,116	187	206	.0231	316	342	454	492
65	8,030	173	192	.0215	293	318	421	457
68	7,912	158	177	.0200	270	293	385	421
71	7,762	144	161	.0185	248	268	350	385
74	7,575	129	146	.0171	225	242	317	347
77	7,353	115	130	.0157	202	216	284	310
80	7,091	102	115	.0143	180	190	251	273
83	6,788	89	99	.0131	159	164	220	236
86	6,446	77	84	.0120	136	140	189	201
89	6,054	67	70	.0110	111	116	163	166
92	5,608	57	56	.0101	95	93	138	133
95	5,101	48	43	.0093	79	71	114	103
98	4,530	39	31	.0085	62	52	91	74
101	3,880	29	21	.0076	47	34	68	50
104	3,142	19	12	.0061	29	20	41	29
107	2,290	10	5	.0043	14	9	20	12
110	1,276	1	1	.0010	2	2	3	2

determined, we get by least squares, $l = 111.4$. Now the portion of the specimen under the clamp is 1 cm. long, and a certain amount of flexure undoubtedly takes place for some distance under the clamp; therefore we might expect the effective bending length to be between 111 and 112 cm.

Taking then the value of l as 111.4, the values of k for the three cases are respectively .000514, .000853, and .001226. The computed values of D are arranged in Table IX. for comparison with the observed values. It will be seen that the agreement is good at every point.

Besides the study of demagnetization, or decrease in the number of lines of force passing through the iron, it is also of considerable interest to investigate the lines of force which escape into the air from the magnetic material.

If we define the *Leakage* as the flux escaping from the metal per centimeter of its length, then it is found that this quantity may decrease at some points of the magnet and increase at others. Of course the leakage as defined may have both positive and negative values. It is measured by sliding the coil over a small constant distance between guides at various points of the magnet, both before and after the flexure takes place. The difference between these two sets of readings gives the *decrease in leakage* occasioned by flexure.

Corresponding to the mode of clamping just discussed the leakage decrease has its greatest value at the position of the clamp ($x = 0$). Its value is zero at $x = 28$, the point of maximum demagnetization; while there is a maximum at $x = 70$ ($\frac{5}{8}l$); and a minimum at $x = 98$ ($\frac{7}{8}l$).

When the clamp is placed at the quarter-length of the magnet, that is at $x = 28$, there is a maximum decrease in leakage beneath the clamp; another at 70, as in the preceding case; a minimum at $x = 14$ ($\frac{1}{8}l$); and another at or near 105 ($\frac{15}{16}l$). The corresponding demagnetization curve is shown in Fig. 4(D). It has a maximum at 42, half-way between the clamp and the point of greatest original flux. At the same point the leakage decrease is, of course, zero. In fact we have in general

$$L = \frac{d\phi}{dx}; \quad L' = \frac{d\phi'}{dx},$$

where L and L' represent the leakage before and after flexure. Also

$$\Delta L = L - L' = \frac{d\phi}{dx} - \frac{d\phi'}{dx} = \frac{dD}{dx},$$

since $\phi - \phi' = D$. Therefore ΔL is zero whenever D is a maximum.

A summary is given for comparison (in Table X.) of the points of maxima and minima in the values of D, R, and ΔL, for four different positions of the clamp. The displacement of the end of the magnet ($x = 112$) was 2.3 cm. in each case, making the curvatures at the clamp proportional to 3, 4, 6, and 8 respectively.

It will be noticed that when the clamp is placed at an aliquot part

TABLE X.

Position of Clamp.	Curvature at Clamp.	D	R		ΔL	
		Max.	Max.	Min.	Max.	Min.
0	3	$\frac{l}{4}$	none	none	$\frac{5l}{8}$	$\frac{7l}{8}$
$\frac{l}{4}$	4	$\frac{3l}{8}$	$\frac{5l}{16}$	$\frac{5l}{32}$; $\frac{13l}{16}$	$\frac{l}{4}$; $\frac{5l}{8}$	$\frac{l}{8}$; $\frac{15l}{16}$
$\frac{l}{2}$	6	$\frac{19l}{32}$	$\frac{5l}{8}$	$\frac{l}{8}$	$\frac{l}{2}$	$\frac{l}{8}$
$\frac{5l}{8}$	8	$\frac{11l}{16}$	$\frac{l}{16}$; $\frac{3l}{4}$	$\frac{3l}{32}$	$\frac{5l}{8}$	$\frac{3l}{32}$

of the length of the rod, the maxima and minima are also at simple fractional parts of the length.

Table XI. shows the actual values of R in the four cases just mentioned.

TABLE XI.

Tool Steel.

Displacement of 2.3 cm. at end, $x = 112$.

x.	Clamp at 0.	$\frac{l}{4}$.	$\frac{l}{2}$.	$\frac{5l}{8}$.	x.	Clamp at 0.	$\frac{l}{4}$.	$\frac{l}{2}$.	$\frac{5l}{8}$.
	R.	R.	R.	R.		R.	R.	R.	R.
2	.06204	.11232	.04423	.00423	59	.02471	.06674	.09034	.04447
5	.06124	.08586	.03612	.00643	62	.02309	.06414	.09532	.05251
8	.05867	.07712	.03313	.00632	65	.02154	.06107	.09802	.06197
11	.05635	.07326	.03180	.00602	68	.02001	.05791	.09890	.07386
14	.05402	.07143	.03132	.00622	71	.01851	.05454	.09858	.08735
17	.05191	.07097	.03143	.00673	74	.01707	.05170	.09726	.09673
20	.04979	.07158	.03202	.00759	77	.01565	.04911	.09521	.10180
23	.04770	.07302	.03302	.00848	80	.01433	.04686	.09264	.10445
26	.04549	.07526	.03442	.00942	83	.01314	.04514	.08978	.10526
29	.04328	.07790	.03628	.01055	86	.01201	.04392	.08672	.10438
32	.04114	.07935	.03857	.01187	89	.01098	.04328	.08358	.10248
35	.03772	.07977	.04129	.01344	92	.01011	.04339	.08042	.09963
38	.03707	.07949	.04452	.01529	95	.00931	.04454	.07724	.90586
41	.03513	.07862	.04835	.01754	98	.00850	.04698	.07400	.09022
44	.03325	.07730	.05288	.02013	101	.00755	.05158	.07088	.08260
47	.03145	.07570	.05831	.02338	104	.00605	.05983	.06782	.07288
50	.02975	.07359	.06479	.02700	107	.00428	.07647	.06489	.05913
53	.02809	.07169	.07268	.03203	110	.00102	.12594	.06050	.02821
56	.02636	.06930	.08227	.03765					

Since the strained portion of the rod is shorter in each succeeding case, for equal displacements of the free end the curvature is, therefore, becoming each time greater at the position of the clamp, and also at

all points in the part under flexure. At the displaced end there is no curvature in any case. (See Col. 2, Table X.)

The effect of this increasing stiffness is shown at $x = 56$, the point of maximum initial flux. However, in the last case the portion under strain is so small as to more than compensate for the increased curvature, and R does not exceed in value that of the other case until the point 74 is reached. This is near the point of maximum D. (F, Fig. 4.)

It may be noted also that the value of R at the position of the clamp is greater in each succeeding case.

An interesting relation is found to hold between the position of the clamp and that of the maximum D, but it fails when the clamp is at the middle (E, Fig. 4). It may be stated thus: The position of the maximum demagnetization is half as far ahead of the clamp as the clamp is distant from the middle of the specimen.

From this it would follow that the maximum D would fall at the same

TABLE XII.

Tool Steel.

Rod supported at ends; loaded at middle.

x.	Displacement of Middle = 0.50 Cm.		Displacement = 0.95 Cm.		Displacement = 1.60 Cm.	
	D.	R.	D.	R.	D.	R.
2	81	.0568	169	.1192	287	.2018
5	106	.0438	235	.0968	374	.1542
8	128	.0390	290	.0885	465	.1418
11	150	.0374	344	.0855	533	.1326
14	174	.0372	397	.0848	610	.1303
17	199	.0378	450	.0855	686	.1304
23	251	.0403	554	.0892	836	.1345
29	305	.0440	658	.0948	982	.1415
35	361	.0483	757	.1012	1,120	.1498
41	413	.0526	847	.1079	1,245	.1586
47	454	.0561	912	.1128	1,344	.1663
53	476	.0582	947	.1157	1,402	.1713
59	476	.0583	946	.1158	1,401	.1715
65	452	.0562	909	.1132	1,338	.1667
71	408	.0526	841	.1083	1,233	.1589
77	355	.0482	747	.1016	1,102	.1499
83	297	.0437	645	.0950	955	.1407
89	242	.0399	541	.0894	799	.1320
95	192	.0376	437	.0856	640	.1254
98	169	.0373	384	.0848	557	.1229
101	147	.0379	332	.0856	478	.1233
104	126	.0402	279	.0886	396	.1259
107	106	.0464	222	.0969	309	.1347
110	82	.0643	152	.1194	212	.1660

distance from the clamp no matter which end of the magnet was displaced.

Two cases of symmetrical curvature were tried. In the first the rod was supported on knife-edges at the ends and bent by displacing the middle; in the second the ends were similarly clamped and the middle displaced.

Tables XII. and XIII. show the values of D and R. The demagnetization curves for the first case resemble hyperbolas, having only one maximum and no points of inflection; those for the second case have a maximum at the same point as the others, and also points of inflection at $3l/16$ and $13l/16$. Both sets of curves are, of course, symmetrical about a vertical axis.

TABLE XIII.

Tool Steel.

Rod clamped at ends; loaded at middle.

x.	Displacement of Middle = 0.42 Cm.		Displacement = 0.65 Cm.		Displacement = 1.60 Cm.	
	D.	R.	D.	R.	D.	R.
2	260	.1828	375	.2639	562	.3959
5	320	.1321	466	.1920	739	.3048
8	361	.1103	526	.1604	858	.2617
11	391	.0971	574	.1428	946	.2352
14	412	.0881	614	.1311	1,012	.2162
17	429	.0815	647	.1230	1,069	.2032
23	457	.0736	707	.1138	1,136	.1828
29	489	.0705	773	.1114	1,260	.1816
35	535	.0716	850	.1137	1,401	.1874
41	586	.0746	929	.1183	1,553	.1978
47	632	.0782	1,001	.1238	1,698	.2108
53	660	.0807	1,048	.1281	1,802	.2203
59	658	.0805	1,043	.1277	1,802	.2206
65	622	.0775	980	.1220	1,701	.2118
71	568	.0732	893	.1150	1,554	.2002
77	511	.0695	804	.1093	1,396	.1898
83	465	.0684	729	.1074	1,253	.1845
89	440	.0726	676	.1116	1,144	.1889
95	423	.0829	630	.1234	1,060	.2077
98	411	.0907	603	.1330	992	.2191
101	393	.1012	569	.1466	923	.2379
104	366	.1163	524	.1667	830	.2642
107	324	.1415	463	.2020	715	.3124
110	262	.2051	372	.2916	531	.4161

In the second case the curvature at the middle point is twice as great as in the first case, for the same displacement. However, the value of D at the mid-point is not nearly twice as great. This is because there are

two points of zero curvature, $l/4$ and $3l/4$, to offset the higher curvatures of the middle and ends.

It is evident that the relative demagnetization will be a maximum also at the middle point; but there are minima in both sets of R curves while there were none for the D curves.

When the ends of the magnet were clamped and the middle displaced, the curvature fell linearly toward the quarter points where it became zero, then rose linearly again to the middle. In the corresponding curves for R the minima fall at these quarter points as might be expected, and the high points are the middle and ends. Nevertheless, the values of R cannot be proportional to the curvature as in the first mode of flexure since R does not become zero, but only a minimum where the curvature is zero. This shows conclusively that original flux at the point and curvature are not the only factors to be considered. It seems clear from the investigation up to this point that the demagnetization

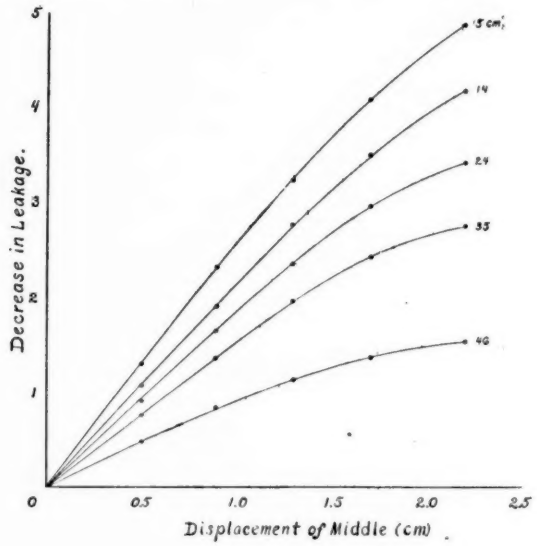


Fig. 5.

taking place at any point is influenced largely also by that at the surrounding points.

When the specimen merely rests at the ends without clamping the minimum values of R occur at $l/8$ and $7l/8$. The reason for these minima is not so clear as before, for the curvature increases linearly from end to middle. The curves for decrease in leakage (ΔL) when the ends of the magnet are supported, show very much the same form whatever the strain; but unlike any of the other modes of flexure it is found that

distinct maxima and minima appear only under small strains. This change of characteristic form is, of course, a gradual one.

When the ends are clamped ΔL becomes a maximum at $11l/32$ and $21l/32$; and a minimum at $3l/16$ and $13l/16$. Such a curve is shown in Fig. 6 (middle curve).

The variation of ΔL with the curvature is represented in Fig. 5 for the case where there is no clamping at the ends. This variation is not the same for all points on the rod, as was the similar variation of R in the torsional case (see Fig. 1), but is greater as the end is approached. Five points are studied in the figure, $x = 5, 14, 24, 35, 46$ cm. respectively, and for each one the curve is found to become more nearly horizontal for large strains.

This is also true for the demagnetization in all cases.

In this instance, however, the relation between leakage decrease and curvature is more nearly a linear one near the ends of the specimen.

IMPACT.

The effect of impact upon a magnetized rod may be observed and measured during the instant it is taking place by means of a coil of wire slipped over the rod, the terminals being connected to a ballistic galvanometer. The coil remains stationary at any chosen position while the blow is struck at any other point desired. The rapid decrease in the flux through the coil produces an induced current whose integral over the interval of the blow is sufficiently large to enable it to be measured with accuracy.

Such a device may also be used to show demagnetization due to the rapid heating of a magnet by a flame, but the effect is immensely smaller.

The hammer used in this series of experiments consisted of a brass cylinder which could be fastened by means of a screw at any point along a brass rod, which in turn swung upon a horizontal pivot.

The energy of the blow could thus be varied in several ways and easily computed.

It was found by the method described above that whenever a fixed blow was delivered repeatedly at the same point on the magnet the demagnetization produced at any particular section grew less and less at each blow. In fact the metal rapidly approaches a state in which it is no longer demagnetizable by the blow which had been operating upon it. However, it is by no means completely demagnetized, and a heavier blow will at once show its effect. It may be stated here that it is not possible, seemingly, to render a magnet neutral by means of torsion, flexure, or impact.

In order to study the effect of longitudinal vibration in the magnetized rod, free from lateral vibration, the magnet was grasped tightly in the hands and struck a solid blow by the hammer at one end and lengthwise of the rod. The original flux and the resulting demagnetization for such an impact are shown in Table XIV. It may be seen at once that the demagnetization is considerably greater at the end struck than at the other end. This would seem to show that the compression wave travelling along the rod is considerably damped in traversing the length of the specimen.

In order to find an equation for this curve it was assumed that D decreases linearly with x; that is that D is proportional to $(k - x)$, where k is a constant greater than the length of the magnet. We may then write $D = c\phi(k - x)$, where ϕ is the initial flux, and c is a constant.

TABLE XIV.

Tool Steel.

Impact (blow at end $x = 0$).

x.	ϕ.	D Observed.	D Computed.	x.	ϕ.	D Observed.	D Computed.
2	1,296	143	175	59	8,533	980	990
5	2,572	332	323	62	8,476	971	975
8	3,349	476	445	65	8,387	957	956
11	4,159	588	548	68	8,260	935	934
14	4,864	675	636	71	8,101	907	908
17	5,484	744	712	74	7,906	875	874
20	6,025	799	776	77	7,675	840	845
23	6,497	842	830	80	7,399	802	807
26	6,912	876	877	83	7,082	761	766
29	7,266	903	915	86	6,719	717	720
32	7,572	924	946	89	6,301	669	669
35	7,831	941	970	92	5,826	616	613
38	8,047	954	989	95	5,282	556	550
41	8,220	964	1,002	98	4,666	489	482
44	8,355	970	1,010	101	3,960	411	405
47	8,456	976	1,014	104	3,122	318	316
50	8,522	980	1,014	107	2,189	211	219
53	8,554	983	1,009	110	1,089	89	108
56	8,559	984	1,001				

When determined by least squares the values of these constants are $k = 412$; $c = .0003286$. From these constants the fourth column of Table XIV. was computed, and it is seen to compare very well with the column of observed values of D. The maximum variation is not over 4 per cent. of the maximum value of D, and over a large portion of the curves it is not over one half per cent.

The form of the leakage decrease curve when a blow is struck at the

middle of a magnet which is not allowed to vibrate transversely is shown in Fig. 6 (*A*). The value of ΔL is nearly a linear function of x, except near the ends.

When a similar blow is struck with the rod clamped only at the ends the resulting curve for ΔL has maxima and minima, and resembles one of the forms discussed under flexure. Nevertheless the two are not alike. This suggested the possibility of analyzing the general impact effect into components, each of which should represent a distinct demagnetizing effect.

In the first place the magnet was clamped throughout its length, except for a few centimeters near the middle, to a heavy, smooth stone slab. A blow of known strength was then delivered at the middle of the specimen, This method of clamping allows no appreciable vibration

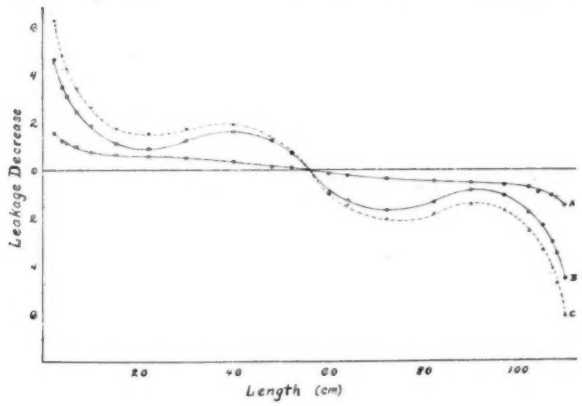


Fig 6.

of the rod, and the level, smooth surface enables the magnet to be securely fastened without any demagnetization due to flexure. In fact the demagnetization produced in this case can be traced solely to the compression wave which emanates from the point of impact of the hammer and spreads out over the entire length of the rod.

The curve for ΔL in this case is shown in Fig. 6 (*A*).

Next the specimen was completely demagnetized by an alternating current and remagnetized with the same initial distribution as in the first case. The ends of the rod only were clamped this time, and the same blow struck at the middle; the rod being allowed afterward to vibrate as a whole. The magnitude of the depression of the middle of the specimen was determined by taking the impression of the middle of the rod, when the blow was delivered, in a block of wax. This deflection amounted to .68 cm. (See Fig. 6, points marked x.)

Again the magnet was completely demagnetized and then brought to the same initial state as in the former cases. It was clamped as before at the ends; the mid-point was depressed by .68 cm., but without any blow; then released, and the rod allowed to vibrate as a whole. The leakage decrease curve thus produced is shown in Fig. 6 (*B*).

Now the sum of curves *A* and *B* is given by the dotted curve *C*, and it is seen that it passes very accurately through the set of points marked by the *x*'s.

This seems to prove rather conclusively that, in general, demagnetization by impact is due to two distinct effects. The predominant one is caused by vibration of the magnet either transversely or longitudinally, particularly the former. The second effect in importance is that of the wave of compression set up by the hammer at the point of impact.

The same proof in terms of demagnetization is exhibited in Table XV.

TABLE XV.

Tool Steel.

Impact (blow at middle).

x.	Blow at Middle. Rod Clamped to Stone Slab. (No Flexure.)	No Blow; Vibration Only. Clamped Ends; Mid-point Deflected, and Rod Vibrates.	Sum of Columns 2 and 3.	Blow at Middle. Clamped Ends.
2	253	625	878	887
5	291	732	1,023	1,031
8	320	806	1,126	1,135
11	343	864	1,207	1,215
14	363	901	1,264	1,275
17	383	936	1,319	1,328
20	400	965	1,365	1,374
23	418	988	1,406	1,417
26	432	1,017	1,449	1,460
29	446	1,045	1,491	1,506
32	461	1,083	1,544	1,558
35	472	1,123	1,595	1,613
38	484	1,169	1,653	1,671
41	492	1,215	1,707	1,728
44	501	1,259	1,760	1,765
47	507	1,299	1,806	1,823
50	513	1,331	1,844	1,860
53	516	1,354	1,870	1,886
56	516	1,362	1,878	1,895
59	516	1,351	1,867	1,881
62	510	1,325	1,835	1,849
65	504	1,287	1,791	1,809
68	498	1,244	1,742	1,757
71	490	1,195	1,685	1,702

It would be difficult to say what would be the form of the analytic expression which would describe accurately and in general the final distribution of magnetic flux in a magnet under strain. Nevertheless, there seems to be sufficient evidence brought out in this investigation to lead us to suppose that such an expression would depend fundamentally upon the initial flux and the curvature at each point.

The experiments, particularly those involving flexure where only a part of the specimen was under strain, show conclusively that curvature does not in general enter the expression as a direct factor; otherwise there would be no demagnetization at points of zero curvature.

No doubt it will be necessary before arriving at a complete solution of the problem to know the general conditions for the equilibrium of lines of force.

The above investigation was carried out in the physical laboratory of Clark University, at the suggestion and under the direction of Professor A. G. Webster, to whom my thanks are due for his interest and assistance.

A TEST FOR X-RAY REFRACTION MADE WITH MONOCHROMATIC RAYS.

BY DAVID L. WEBSTER AND HARRY CLARK.

ONE of the best-known properties of X-rays, and the most important for their application to radiography, is their ability to penetrate all known substances without being refracted. This has been verified by Roentgen and others, and was for a long time accepted as a good argument for the pulse theory in a form involving the assumption that atoms contain no oscillators whose natural frequencies are those of X-rays. Since we now know that this assumption is incorrect, the reason for the absence of refraction is not evident. Chapman[1] seems to have been the first to appreciate this. He tried a prism of ethyl bromide vapor, using heterogeneous rays which included the characteristic frequencies of bromine, but obtained no refraction, although a refractive index of $1 \pm .0003$ could have been detected. Similar results were obtained in a more exact experiment on potassium bromide prism by Barkla,[2] who could have detected an index differing from 1 by only .000005.

These results show conclusively that rays of most frequencies are refracted very little, if at all. The appreciable refractive effect of a resonance frequency in optical spectra does not always extend far along the spectrum, however, and if this is true of X-rays, the refracted part of a heterogeneous beam might contain so small a part of the energy and be so widely dispersed as to escape detection. This paper presents an analysis showing in what part of the spectrum the refraction, if it exists, ought to be greatest; and showing that under reasonable conditions this greatest value should be extremely small. It then describes experiments made to test the theory. Monochromatic rays were used in these experiments. No evidence of refraction was found.

ANALYSIS OF CONDITIONS FOR REFRACTION.

As a starting point for this analysis we may take the formulas for the refraction and absorption of electromagnetic waves in matter given by Lorentz, Theory of Electrons, Chap. IV., which we may write in the form

[1] Chapman, Proc. Camb. Phil. Soc., p. 574 (1912).

[2] Barkla, Phil. Mag., Apr., 1916.

$$\mu(n) = 1 + \sum_i \frac{2\pi N_i e_i^2 m_i (n_i^2 - n^2)}{m_i^2 (n_i^2 - n^2)^2 + n^2 g_i^2}, \tag{1}$$

$$k(n) = \frac{1}{c} \sum_i \frac{4\pi N_i e_i^2 n^2 g_i}{m_i^2 (n_i^2 - n^2)^2 + n^2 g_i^2}. \tag{2}$$

These differ in several respects from the forms given by Lorentz. They express the summation of effects due to different resonance frequencies. Ordinary electrostatic units are used. k represents the logarithmic decrement of intensity rather than of amplitude. $\mu(n)$ and $k(n)$ are the coefficients of refraction and absorption respectively at the "radian frequency" n, N_i the number of electrons with resonance at n_i, g_i their damping coefficient, e_i and m_i their charge and mass.[1]

These formulas give the results, familiar in the anomalous dispersion of light, that the strongest refraction occurs on the sides of the absorption bands, and that any resonance frequencies in or near the visible can have no great effect on X-rays. This latter result is confirmed by Chapman's experiments and those of Barkla.

To apply these formulas to an actual case we must make use of the resonance frequencies given by the absorption spectrum (not the fluorescent emission spectrum), and from them predict the refraction. This prediction, which would seem at first sight to involve many doubtful assumptions, may be made without ambiguity by the use of the approximate equation,

$$\mu(n) - 1 = \frac{c}{\pi} \int_0^\infty \frac{k(n_j) dn_j}{n_j^2 - n^2}, \tag{3}$$

which connects the values of μ and k given in (1) and (2).

To prove this formula, the value of $k(n_j)$ may be taken from equation (2) and in each term of the sum the variable of integration may be transformed to $x = n_j^2 - n_i^2$. The approximations used in the integration are as follows: (*a*) n is substituted for a factor n_j (to the first power) in the numerator of the integrand; (*b*) n^2 is substituted for the n_j^2 appearing as a coefficient of g_i in the denominator; (*c*) the lower limit of the integration with respect to x is shifted from $-n_i^2$ to $-\infty$. The errors introduced by the first two approximations cannot readily be evaluated, but are certainly small unless g_i is large. This is shown by the fact that if n_i is substituted, instead of n, in each case, the result will be only a similar change in the g_i term in the value of $\mu(n)$. The error

[1] Of course the conditions of the X-ray problem do not permit a rigorous use of Lorentz's "physically infinitesimal" elements of volume and the analysis based on them, but since the retardation and absorption per wave length are very small, it seems improbable that the rays are deformed by single atoms enough to make any error in this analysis in other problems than those of scattering.

introduced by (*c*) consists in dropping some small additive terms, all of which contain $g_i/m_i n$ as a factor. Therefore it also is very small if the influence of any one resonance frequency does not extend far along the spectrum.[1]

For the present case, using the results of Duane,[2] Siegbahn,[3] Bragg,[4] and Hull and Rice[5] we may describe the X-ray absorption spectrum of any element by the formula

$$k(n) = A_M \left(\frac{n_M}{n}\right)^3 + A_L \left(\frac{n_L}{n}\right)^3 + A_K \left(\frac{n_K}{n}\right)^3. \qquad (4)$$

A_K shall be zero at all frequencies below n_K, that of the eight-fold rise in k (n) at the K series of the absorbing substance, and constant for all frequencies well above this. A_L is similarly defined with reference to the four-fold rise at the L series, while A_M is constant for all ordinary frequencies. This form of representation is derived from the fact that each rise is closely related to the phenomena of fluorescence of the corresponding series. Since it is improbable that there is any actual discontinuity in nature, we shall assume the rise in A_K to be continuous through a very short range n_K to n_K'. The exact value of n_K for the case of rhodium appears from our preliminary photographs to be that of the $K\gamma$ line itself; and the rise occurs so suddenly that no definite value can be assigned to the difference $n_K' - n_K$, but it is probably not much over 0.001 n_K and may be very much less. Wagner[6] obtained similar results using palladium and other metals.

Applying (3) to this spectrum, and considering only the refraction due to oscillators associated with the K series, and neglecting the fraction $(n_K' - n_K)/(n - n_K)$ we derive the following expression:

$$\{\mu(n) - 1\}_K = -\frac{A_K \lambda_K}{4\pi^2}\left\{\frac{n_K^2}{n^2} + \frac{n_K^4}{n^4}\log\left|1 - \frac{n^2}{n_K^2}\right|\right\}. \qquad (5)$$

[1] It is interesting to notice that we can obtain some information about these quantities for the case of a continuous spectrum containing oscillators resonating to any frequency by substituting an integration for the summation in (1) with $f(n_i)dn_i$ for N_i, and obtaining the approximate equation

$$k(n) = \frac{2\pi^2 e^2}{cm} f(n),$$

which is independent of g_i. This integration, however, involves the approximation (*c*) of (3) and also the assumption that e_i, m_i, g_i, and $f(n_i)$ do not vary much within the region of the spectrum absorbed by oscillators of any one resonance frequency. This equation, combined with an approximate form of (*c*), in which $g_i = 0$, gives (3), but with these unnecessary restrictions.

[2] Duane, Amer. Phys. Soc., Dec., 1914, stating results of calculation from absorption measurements by Barkla and Sadler and wave-length measurements by Moseley.

[3] Siegbahn, Phys. Zeitschr., Aug. 15, 1914.

[4] Bragg, Phil. Mag., March, 1915.

[5] Hall and Rice, PHYS. REV., Sept. 1916.

[6] Wagner, Ann. d. Phys., March, 1915, p. 868.

Now since $A_K\lambda_K$ is the increase at λ_K of the fraction of the rays absorbed in going a single wave-length, it is very small. For rhodium, for example, it is about 3.5×10^{-5}. Consequently the refraction cannot be measurable unless n has a value so near that of n_K that the logarithm is very large. For such a value we cannot neglect the difference $n_K' - n_K$. Letting $p = (n_K' - n_K)/n_K$ and $q = (n - n_K)/n_K$, and neglecting all terms but the logarithms, we find

$$\{\mu(n) - 1\}_K = -\frac{A_K\lambda_K}{4\pi^2}\left\{\frac{p-q}{p}\log|p-q| + \frac{q}{p}\log|q|\right\}. \qquad (6)$$

This has a maximum at the middle of the range n_K to n_K', and is nearly equal to

$$\frac{A_K\lambda_K}{4\pi^2}\log\frac{1}{p}$$

throughout this range. In this part of the spectrum the terms due to the L and M series may be shown by a brief calculation, confirmed by Barkla's experiment, to be very small. Consequently greatest refraction ought to be found at the discontinuity in the absorption, that is, at the $K\gamma$ line of the refracting substance. It can scarcely be measurable, however, unless the value of p is extraordinarily small, or unless this theory has a fundamental defect. We must therefore expect a negative result in the following test for refraction.

EXPERIMENTAL TEST FOR REFRACTION.

In these experiments we used the rays from the rhodium target of a Coolidge tube operated by a storage battery at about 40,000 volts. This voltage was not high enough to give general radiation of such frequency that its second-order spectrum could confuse the results.[1] The spectrum was obtained by reflection from the (100) face of a crystal of calcite. A small rectangular cell of brass, in which a prism of rhodium was mounted as shown in Fig. 1, was placed in the path of the reflected rays and behind this at some distance was placed a photographic plate. The brass cell being opaque to the rays cast a shadow upon the plate. It was not, however, sufficiently wide to obscure the rays completely, so the photographs show the ends of each spectrum line projecting above and below the shadow.

Fig. 2 shows the general appearance of a photograph with the projecting ends of the spectrum lines at a, a. The prism faces were carefully ground and were plane down to within a very short distance from their

[1] Duane & Hunt, Amer. Phys. Soc., April, 1915, PHYS. REV., Aug., 1915. Hull, Amer. Phys. Soc., Nov., 1915, PHYS. REV., Jan., 1916. D. L. Webster, Proc. Nat. Acad., Feb., 1915, PHYS. REV., June, 1916, p. 599.

line of intersection. The cell was so adjusted that no rays were allowed to reach the prism[1] at a point where it was less than .06 mm. thick. At this place the faces were nicely plane. This limiting position will be referred to as the *working edge* of the prism.

Although rays fell on nearly all parts of the prism, very few were transmitted save near its *working edge*. With a properly timed exposure only a very narrow line was recorded opposite the prism, as shown at *b*,

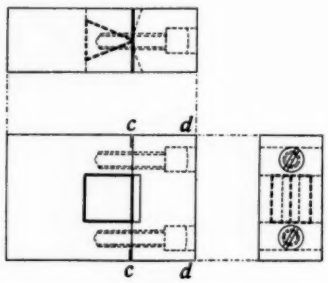


Fig. 1.

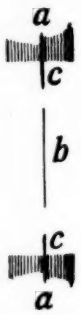


Fig. 2.

Fig. 3.

Fig. 2, and so it was unnecessary to use any slits to determine the width of the transmitted ray. Since an increase in the time of exposure would have made the line broader, it was faintly recorded on the plate at best. For this reason it would have been difficult to reproduce the photographs themselves in print.

A pair of slots, *c*, *c*, Fig. 1, about .1 mm. wide were cut in the brass cell so that one edge of each slot was in line with the *working edge* of the prism. The rays which came through these slots fell upon the plate, as shown at *c*, *c*, Fig. 2. These marks at *c*, *c* showed by comparison with *a*, *a* just what part of the spectrum had been transmitted, and by their alignment with line *b*, whether or not the transmitted rays had been refracted.

To test the regularity of the reflecting crystal many of the photographs taken as described above were followed by others in which the cell was withdrawn toward the left by a micrometer screw through such a

[1] Barkla (note 2 above) worked with potassium bromide prisms the faces of which were cleavage planes. That such surfaces are more perfectly smooth than ground ones is not quite evident, for these crystals are usually very irregular. It should be noted, however, that a very smooth surface is not essential when working with X-rays. A surface is generally said to be *optically polished* when its irregularities are small compared to the wave-length of light. To speak strictly, however, it is not the surface of the substance, but rather the transmitted or reflected *wave* whose irregularities must be small. The refractive effect of any substance for X-rays, if it exist, is certainly very small and as a result the waves are but slightly distorted by comparatively great irregularities of the surfaces through which they are transmitted.

distance that the edge d, d (Fig. 1) stood exactly in the position formerly occupied by the working edge of the prism. The result is indicated in Fig. 3. The edge d, d of the shadow of the cell was found to be straight in all cases.

The sharpness of the photographs makes it reasonable to assume that a deviation of the transmitted ray amounting to as little as .2 mm. or even .1 mm. could have been detected. With the plate 80 cm. from the prism, as we used it in most of the trials, this would correspond to a refractive index of 1.0004 or 1.0002. Many trials were made, using various parts of the spectrum, including several parts of the γ line, the β line, both members of the α doublet, and the general radiation between β and γ. In none of these was there any trace of refraction.

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THERMIONIC CURRENTS FROM MOLYBDENUM.

BY E. R. STOEKLE.

IN one of his early papers[1] O. W. Richardson developed the theory of the nature of the discharge of negative electricity from hot metals by a quantitative study of the relation between the temperature and the thermionic saturation currents from platinum, carbon and sodium. Assuming that the free electrons of a metal have their velocities distributed according to Maxwell's law and that they are in thermal equilibrium with the molecules, Richardson arrived at the following expression for the saturation electron current flowing from a hot metal to a cold electrode:

$$I = AT^{\frac{1}{2}}e^{-\frac{b}{T}}, \qquad (1)$$

where T is the temperature in degrees K. and A and b are constants. In a further theoretical investigation,[2] he assumed that the constant b depended upon the temperature in such a manner as to make the more accurate expression for the current:

$$I = CT^{2}e^{-\frac{a}{T}}.$$

These two expressions are, however, practically identical over all ranges of temperature for which the thermionic currents from metals can be measured. This is true because of the overwhelming importance of the inverse exponential factor.

It has been shown by many writers that the early thermionic experiments were profoundly influenced by the effects of the residual gases and vapors in the presence of the hot metals. H. A. Wilson,[3] experimenting with platinum showed that the negative thermionic currents could be greatly diminished by treating the metal with nitric acid. Since the presence of hydrogen increased the currents, Wilson concluded that the diminution observed was due to removal of occluded hydrogen from the metal by the nitric acid and consequently was inclined to attribute the emission of electrons to this occluded gas. Richardson, however, explained the diminution as being caused by the formation of a

[1] Camb. Phil. Proc., Vol. XI., p. 286 (1901). Phil. Trans., *A*, Vol. 201, p. 497 (1903).

[2] Phil. Mag., Vol. 23, 1912, p. 594. Phil. Mag., Vol. 23, 1912, p. 263.

[3] Phil. Trans., *A*, Vol. 202, p. 243.

double layer upon the surface of the platinum, thus hindering the escape of the free electrons from the metal.

More recently Pring and Parker[1] believed that they had shown that the negative currents from pure carbon could be made vanishingly small by long-continued heating; but Richardson[2] has shown that the magnetic field due to the large heating currents used by these observers exerted a force upon the electrons sufficient to deflect most of them from their path to the cold electrode.

R. Ichinohe and M. Kinoshita[3] investigating the negative currents from platinum carbon and tungsten found that Richardson's formula did not hold for high temperatures due to the ionization caused by the collision of the emitted electrons with the residual gas molecules in the apparatus. This collision ionization is accompanied with the appearance of a blue glow in the space surrounding the hot metal.

Another attempt to disprove the existence of a true thermionic emission from metals and to attribute it to the action of gases and other impurities in the metals was made by Fredenhagen.[4] He performed experiments on molten sodium which had been purified by repeated distillation in vacuo. After continuously heating the sodium in vacuo for intervals as long as 58 hours, he obtained electron currents from the molten surface which were vanishingly small compared to those obtained by Richardson, thus proving that the thermionic currents from sodium were largely dependent upon the purity of the metal.

In defence of the existence of a true thermionic emission, Richardson[5] performed some experiments on tungsten under such good vacuum conditions that he was led to conclude that the electron emission from hot tungsten could not originate from any process involving the consumption of the hot filament. His conclusions, however, as to the absence of any appreciable change in the electron emission even upon allowing "considerable changes in the amount and nature of the gases present in the tube" are not in accord with the observations of I. Langmuir upon the thermionic emission from tungsten nor with the results of the experiments upon molybdenum to be set forth in this paper.

The numerous experiments performed upon tungsten by I. Langmuir,[6] S. Dushman,[7] and K. K. Smith[8] all seem to point to the existence of a

[1] Phil. Mag., Vol. 23, p. 192 (1912).
[2] Proc. Roy. Soc., *A*, Vol. 90, p. 174.
[3] Coll. of Sci. and Eng. Mem. Kyoto, 3, p. 171 (1911).
[4] Verh. d. Deutsch. Phys. Gesell., 14, p. 384 (1912).
[5] Phil. Mag., Vol. 2, ser. 2, p. 450 (1913).
[6] PHYS. REV., Vol. 2, ser. 2, p. 450, 1913.
[7] PHYS. REV., Vol. 4, ser. 2, p. 121, 1914.
[8] Phil. Mag., Vol. 29, p. 802, 1915.

true electron emission from tungsten which obeys Richardson's equation over a wide range of temperature.

On account of its high melting point (2530° C.) molybdenum is a suitable metal for the study of negative thermionic discharge. Except for some work done by Langmuir[1] incidental to his careful study of the thermionic emission from tungsten, there has been no investigation of the thermionic behavior of this metal.

In 1912–13 the writer performed in this laboratory some experiments upon the thermionic discharge from molybdenum which gave results approximately in accord with Richardson's theory over a range of temperature from 1200° C. to 2200° C. The currents obtained in these experiments were however affected by the occurrence of collision ionization at the higher temperatures and also probably by changes in the molybdenum surface due to the interaction of the molybdenum with the residual gas. Since the results of these experiments were not published at the time they were performed, a brief summary will be interesting, especially as a means of showing the effect of the improved vacuum obtained in the subsequent experiments.

Temperature Measurements.—In all of the experiments to be described the temperature of the incandescent wire was measured through a plate-glass window by an optical pyrometer of the Holborn-Kurlbaum type, used in the manner described by Mendenhall,[2] and calibrated by observations on the melting points of gold and palladium in a standard black-body furnace. These temperatures were then corrected for the absorption and reflection by the glass window on the apparatus, and reduced from "black-body" to true temperature by means of the data given by Mendenhall and Forsythe.[3]

The following will indicate the accuracy of the calibration of the pyrometer. This calibration consists in determining the current necessary to heat a small lamp to the same monochromatic brightness as that of the black body source whose temperature is known. The comparison lamp could be matched to several known intensities determined by interposing rotating discs of known sector aperture between the pyrometer and the black-body source, which was held at a known constant temperature by means of a thermocouple. By Wien's distribution law the temperatures corresponding to these intensities were determined and a curve plotted showing temperature as a function of the corresponding lamp currents. Two such curves were plotted, one with the black body source at the gold melting point (1063° C.) the other with the source

[1] Loc. cit.

[2] PHYS. REV., 33, p. 74, 1911.

[3] Astrophys. Jour., 37, p. 380, 1913.

at the palladium melting point (1549° C.). It was found that these two curves overlapped to within 2° C. in the neighborhood of the gold melting point. Furthermore a temperature of 1326° C. was established on the furnace by adjusting its temperature until the pyrometer indicated the gold point when a 1/15 sectored disc was interposed. This temperature fell within 3° of the curve established from the palladium melting point, thus indicating an accuracy within 3° at 1325° C. Taking into account further probable errors due to the reduction of black-body to true temperature and perhaps in the determination of the correction for the glass window, a reasonable estimate for the accuracy of the measurements of the temperature of the molybdenum wire would be about 5° at 1500° C. and 10° at 2000° C.[1] This is the estimated error in the absolute value of the temperature, but the accuracy with which relative temperatures could be measured was of the order of one degree in 1000° and 3° in 2000°, since the pyrometer could be set and would hold its calibration to this degree of accuracy.

THERMIONIC CURRENTS IN PRESSURES ABOVE .001 MM. OF MERCURY.

These experiments were performed in a glass apparatus evacuated by a Gaede mercury pump. No precautions were taken to exclude vapors from the apparatus except the use of phosphorus pentoxide as a drying agent. The pressures were measured by a McLeod gauge. The pressure in the apparatus always rose throughout an experimental run from about .001 mm. at 1400° C. to as much as .025 mm. at 2200° C. This rise was due to the liberation of adsorbed gases from the glass walls of the vacuum vessel, and especially from the cylindric electrodes surrounding the incandescent molybdenum wire. This wire was .5 mm. in diameter and 8 cm. long. The electrodes consisted of a central cylinder which collected the electrons emitted by the uniformly heated central portion of the wire, and of two coaxial cylindric guard rings which removed the electrons emitted by the cooler ends of the wire. The saturation electron currents flowing to the central cylinder were measured by means of a galvanometer equipped with shunts to vary the sensibility, and the larger currents were measured by a mil-ammeter. The circuit is shown diagrammatically in Fig. 1, *G* being the galvanometer and

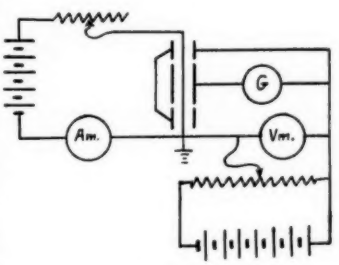

Fig. 1.

[1] The effective transmission of the red glass used was assumed not to shift with the intensity which is not however strictly true as shown by Hyde, Cady and Forsythe, Astrophys. Jour., Vol. 42, p. 294 (1914). This may account for the discrepancies observed.

Am and *Vm* respectively an ammeter to measure the heating current, and a voltmeter to measure the P.D. between the filament and cylinder. This type of circuit was used in all the experiments to be described in this paper.

The results of a number of runs in this apparatus are typified by the Curves I. and II. of Fig. 2 in which $\log_{10} I - \frac{1}{2} \log_{10} T$ is plotted as a

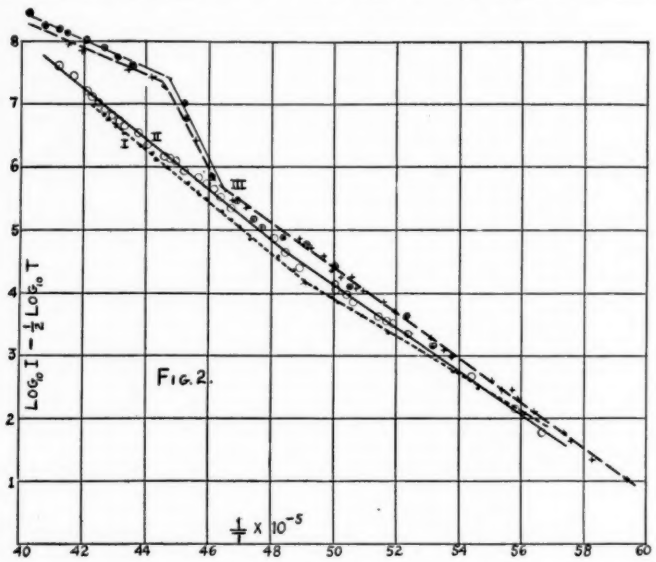

Fig. 2.

function of $1/T$, where I is the saturation current in amperes per square cm. It is evident that if the experimental results are in accord with Richardson's equation eq. (1), these curves should be straight lines whose slopes when multiplied by the modulus 2.303 should give the constant b. In fact it was found that in every case the results could be represented by two straight lines intersecting between 2000° K. and 2100° K. The temperature at which the discontinuity in slope occurred was in general higher the lower the pressure in the apparatus, although no quantitative conclusion on this point was possible because of the difficulty of controlling the pressure independently of the temperature.

The most probable explanation of this change in slope is the occurrence of ionization by the collision of the residual gas molecules with the emitted electrons and the effect of the gas on the surface of the wire. An increase of the temperature of the wire increased the number of collisions of electrons with gas molecules not only because of the increase of the space density of electrons but also because of an increase in the pressure of the gas caused by additional heating of the surrounding parts. In

spite of these disturbing factors the curves conform approximately to Richardson's equation over a range of temperature extending from 1700° K. to 2475° K. giving the following average values of the constants

$$A = 5 \times 10^{11} \text{ amp. cm.}^{-2} \text{ deg.}^{-1/2},$$

$$b = 8.1 \times 10^{4} \text{ deg.}$$

For the four experimental runs taken the values of b were 82,000; 83,800; 79,500 and 77,500 the first two values being obtained from the lower Curves I. and II. of Fig. 2.

Another interesting point found in these experiments was that a layer of molybdenum oxide on the surface of the wire greatly increased the thermionic emission. The wire was oxidized while at about 1200° C. by the accidental admission of air into the apparatus. The presence of a discontinuous layer of oxide could be detected through the pyrometer telescope by the dark and bright areas caused by the difference in the emissivity of the metal and the oxide surface. The two curves of Fig. 3

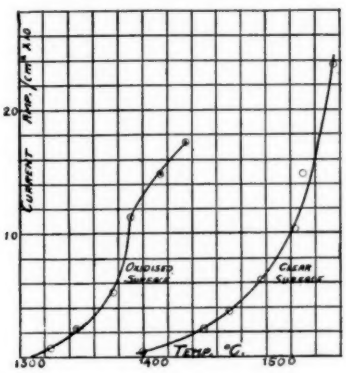

Fig. 3.

show the results of an experiment on such an oxide surface. The pressure in the apparatus was .012 mm. and a potential difference of 80 volts, enough to saturate the thermionic current, was applied between the wire and the cylindrical anodes. The curve to the left shows the current rising very rapidly with the temperature up to about 1400° C., where it begins to fall off rapidly with time until it reached the small value shown by the right-hand curve. At the same time that this decay of the current took place the bright streaks disappeared rapidly from the surface as viewed through the pyrometer. When the surface had cleared the current assumed a steady value and increased in the usual way with the temperature as shown by the curve to the right. This experiment shows that a molybdenum surface visibly coated with oxide

has a much larger emissivity than the clear metallic surface. The pressure in the apparatus throughout this experiment was so high that a continual chemical action of the gas on the hot molybdenum surface may have liberated electrons in the manner studied by Haber and Just. Further experiments on the thermionic activity of an oxidized molybdenum surface especially in much higher vacua would be interesting.

FURTHER EXPERIMENTS WITH IMPROVED VACUUM.

In order to minimize the liberation of adsorbed gases from the parts of the apparatus heated by the radiation from the incandescent wire a tube was designed in which these parts were water cooled. The tube T, Fig. 4, was made of thin glass and was of about 2.8 cm. bore. The inside was silvered, except for a small collar at the top and bottom. These collars insulated the silver surface from the heavy brass caps K which led the heating current to the .5 mm. molybdenum wire f. The silver surface acted as the anode and communicated with the outside through the platinum seal P. The emission from the cooled ends of the wire was eliminated by the shields S. Water was circulated through the space W and thus was very effective in keeping the anode and the ends of the apparatus cool. The spring R maintained a slight tension on the wire and kept it from bending out of alignment at the high temperature.

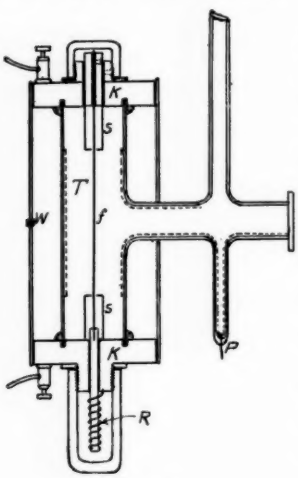

Fig. 4.

This apparatus was connected by one-inch glass tubing to a Gaede mercury pump and it was possible to keep the pressure throughout an experimental run within the limits .0004 mm. to .0026 mm. A number of runs were made with different molybdenum wires and the thermionic saturation currents per square cm. were reproducible with considerable accuracy at low temperatures. The results of experiments on three wires are shown in the curves of Fig. 5, which show good agreement up to a temperature somewhat above 1700° C. At this temperature the pressure began to rise rapidly to values above .001 mm., this accounts for the irregularities of the thermionic current shown in the curves.

The application of Richardson's equation to these curves is however of greater interest. The two upper curves III. of Fig. 2 show the quantity $\log_{10} I - \frac{1}{2} \log_{10} T$ plotted against $1/T$ for two of the wires. These curves are seen to be approximately in accord with Richardson's equa-

tion up to a temperature of 1900° C., giving for the constants

$$A = 4.0 \times 10^{12},$$
$$b = 8.48 \times 10^{4}.$$

Between 1900° C. and 2000° C. they show however a very marked change in slope. Beyond 2000° C. the slope has reached a value which gives

$$A = 10^{8},$$
$$b = 5.5 \times 10^{4}.$$

Numerous curves were plotted showing the relation between the thermionic current and the potential of the anode at various temperatures up to 2150° C. These curves all showed saturation of the current

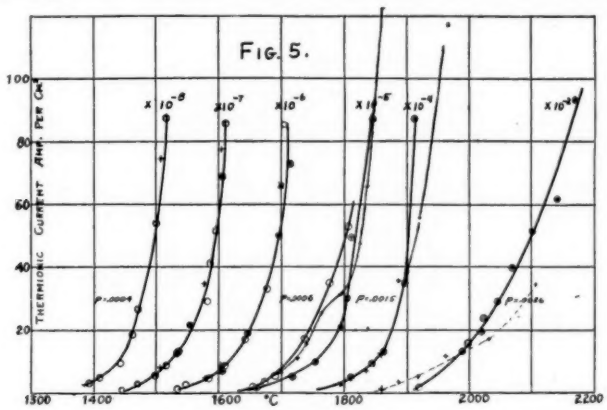

Fig. 5.

in the neighborhood of 20 volts. At the higher temperatures currents of the order of 1 ampere per square cm. of surface or about .12 ampere, per cm. of length of the wire were measured. Langmuir[1] has shown that pure electron currents of this magnitude should vary as the 3/2 power of the potential; and in view of this fact the currents measured should only have reached saturation at a potential of the order of 400 volts. Since these conditions did not hold the currents measured must have been influenced by the occurrence of collision ionization. To remedy these difficulties a new type of water cooled apparatus was designed, and evacuated with a molecular pump which became available at this time.

EXPERIMENTS WITH AN IMPROVED WATER-COOLED APPARATUS EVACUATED BY A MOLECULAR PUMP.

In order to further improve the vacuum especially by preventing the liberation of occluded gases at the high temperatures the water-cooled

[1] Loc. cit.

apparatus, shown in cross section in Fig. 6, was constructed. The anode which collected the electrons emitted by the incandescent wire consisted of the brass cylinder A, which surrounded the wire. It was equipped with a side tube and window W for observing the temperature of the wire. The water jacket J ocmpletely surrounded the anode with water

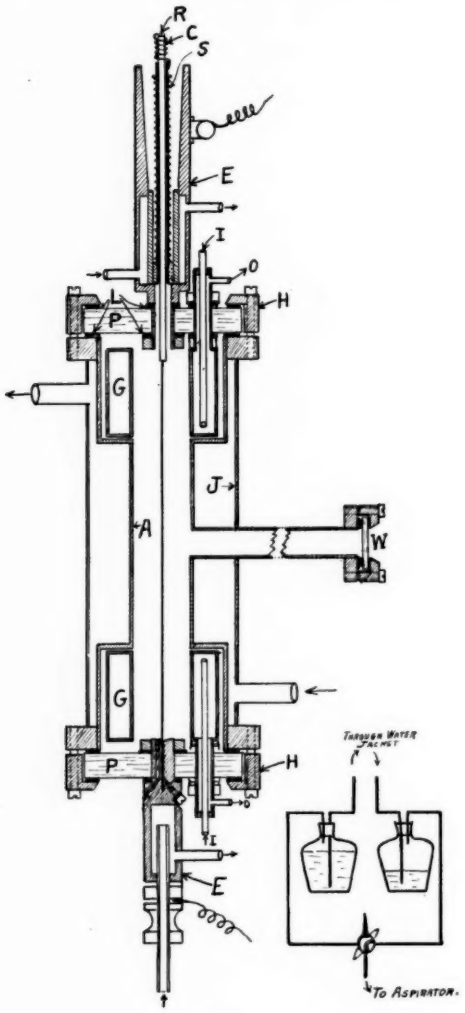

Fig. 6.

supplied by an insulated circulating system. Such a system was easily constructed of two large glass bottles connected together through the jacket J, and joined to a water aspirator by means of a stopcock in such a manner that the air could be withdrawn from either bottle, thus causing

the water to flow from one to the other through the jacket *J*. The scheme is shown in the small diagram of Fig. 6. The guard rings *G* served to collect the electrons emitted by the ends of the hot wire and were cooled by water flowing in at *I* and out at *O*, the circulation being accomplished by another insulated system.

The upper and lower electrodes *E*, which carried the current to heat the filament were also water cooled by connecting them directly to a faucet, thus also connecting the filament to ground. This filament which was of .5 mm. molybdenum wire was clamped to the lower copper electrode by a set screw as shown, and the hole for this screw was made airtight by a cap screw and washer sealed with hard wax. The upper electrodes supported the rod *R* by means of the spiral spring *S*, thus keeping the filament, which was clamped to *R*, under tension, and in alignment along the axis of the cylindric anodes. In order to insure good electrical contact the flexible copper conductor *C* connected the upper electrode to the filament support rod *R*. This upper electrode also served as the vacuum pump connection to the apparatus. A glass cone was ground to fit the electrode and the joint made airtight by sealing its outermost part only with hard wax. Thus any vapor arising from this wax would have to travel along the narrow space between the ground surfaces. The amount of any such vapor which could thus get into the apparatus would be extremely minute.

Both electrodes as well as the tubes which supported the guard rings were securely clamped to the glass end plates *P* by means of nuts. Their contacts with the glass plates were made airtight by lead washers *L*. The plates were also tightly pressed by means of the rings *N* against lead gaskets soldered to brass collars on the ends of the tube. In a similar way the window *W* was fastened airtight to the side tube. These glass to lead joints were almost airtight, but to insure perfect tightness they were each surrounded from the outside with hard wax. Without the wax a pressure of .001 mm. could be maintained inside of the tube, so that when the hard wax was applied only a negligible amount of any vapor arising from it would leak into the apparatus.

The apparatus was connected to a "molecular pump" by means of glass tubing 3 cm. in diameter. A liquid air trap containing cocoanut-shell charcoal prevented the passage of vapors to the apparatus and also helped to absorb gases. A single stopcock of the right-angle type with a hollow large-bore piston was used near the pump. It was made airtight by means of a mixture of beeswax and resin instead of either the usual stopcock grease or the rubber grease, both of which were found to be unreliable for high vacuum work.

The McLeod gauge used could be read to 5×10^{-6} mm. of mercury, but it was of course subject to the uncertainties of this type of gauge when used to measure such low pressures.

The $\frac{1}{2}$ mm. molybdenum wire used in these experiments was from a new supply kindly provided by Dr. Langmuir of the General Electric Company's laboratory at Schenectady.

In order to determine whether pure electron currents free from gaseous ionization could be obtained in this apparatus the relation between the thermionic current and the potential of the cylindric anode was studied.

THE RELATION BETWEEN THE THERMIONIC CURRENT AND THE APPLIED POTENTIAL DIFFERENCE IN A HIGH VACUUM.

The relation between the currents conducted through space by ions of one sign and the impressed potential was derived by Child[1] for a current of positive ions, and by Langmuir[2] for a pure electron current. Child's calculation for the case of two infinite parallel planes resulted in the equation

$$I = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\frac{V^{\frac{3}{2}}}{x^2}, \qquad (1)$$

where e/m is the ratio of the charge to the mass of a positive ion, x is the distance between the planes, and V the applied potential difference.

Langmuir gives the corresponding calculation for the flow of electrons between infinite planes, and also between an infinitely long wire and a concentric cylinder. For the latter case he obtains the following expression for the current per unit length of the wire:

$$I = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}}\frac{V^{\frac{3}{2}}}{r}, \qquad (2)$$

where r is the radius of the cylinder which is assumed large compared to that of the wire.

The $V^{3/2}$ law was verified experimentally by Langmuir[3] Dushman,[4] and Lilienfeld.[5] For large current densities, however, Lilienfeld found that the electron current from incandescent CaO and Ta varied as V^2 instead of $V^{3/2}$. The explanation of this deviation from the theoretical $V^{3/2}$ law is not apparent. It may be attributed to the presence of positive ions in the space, or to the complicated boundary conditions in the Lilienfeld experiment, where the electrons were forced to flow through a

[1] PHYS. REV., 32, p. 492 (1911).
[2] Loc. cit.
[3] Loc. cit.
[4] PHYS. REV., Vol. 4, p. 121 (1914).
[5] Ann. d. Phys., 32 (1910); Ann. d. Phys., 43 (1914).

cylindric glass tube, the potential difference between any two points in the electron stream being measured by platinum wires inserted through the walls of the tube. In fact it may be that the current rose faster than the 3/2 power of the voltage because of the emission of secondary electrons from the first of these potential taps, for, as was shown by v. Baeyer and Gehrts, this secondary electron emission increases very rapidly with the voltage when the velocity of the incident electrons exceeds that due to a drop of 30 volts. Dushman[1] however, showed that the $V^{3/2}$ law held with considerable accuracy for a tungsten wire discharging electrons to a 2.5 cm. diameter concentric cylindric anode even when the currents were as large as 120 miliamperes per unit length of the wire.

Since the law ceases to hold as soon as positive ions are formed in the space between the wire and the anode, it serves as a delicate test for the purity of the electron current; and also since the number of positive ions which are formed for a given electron current increases with the pressure in the apparatus the law serves as a criterion for the degree of vacuum attained.

In the apparatus of these experiments there was a radial electric field between the central 8.45 cm. of the hot-wire cathode and the cylindric anode with the exception of a possible very slight asymmetry caused by the insertion of the side tube through which the temperature was observed. The diameter of this side tube was 1 cm. and its length about 14 cm.

Since the equation (2) strictly applies only if the cathode and the anode are equipotential surfaces it is necessary in order to test the validity of the law to correct for the fall of potential along the filament due to the heating current.[2]

Let v be the fall of potential along the length l of the central portion of the wire covered by the central anode cylinder; and let V be the voltage of the battery whose positive terminal is connected to the anode. Now if the negative terminal of V be considered as connected to the negative end of l then the potential of the anode with respect to an element dx of l, distant x from the negative end of l will be $V - (v/l)x$. If on the other hand the negative terminal of V be connected to the positive end of l the potential of the anode with respect to an element dx distant x from the positive end of l will be $V + (v/l)x$.

Hence by eq. (2) the current between the anode and the element dx will be:

[1] Loc. cit.

[2] Mr. W. Wilson has informed me that he made this correction substantially as in the following, in a paper read before the Physical Soc. in Dec., 1914.

$$di = k\left(V \pm \frac{v}{l}x\right)^{\frac{3}{2}} dx,$$

the sign depending on whether the negative of V be connected to the positive or negative end of l. The value of the constant is

$$k = \frac{2\sqrt{2}}{9}\frac{1}{r}\sqrt{\frac{e}{m}}.$$

The total current of electrons from the wire of length l will therefore be

$$\begin{aligned} I &= k\int_0^l \left(V \pm \frac{v}{l}x\right)^{\frac{3}{2}} dx \\ &= kV^{\frac{3}{2}}\int_0^l \left(1 \pm \frac{v}{Vl}x\right)^{\frac{3}{2}} dx \\ &= kV^{\frac{3}{2}}l\frac{2}{5}\left[\pm\frac{V}{v} \pm \frac{V}{v}\left(1 \pm \frac{v}{V}\right)^{\frac{5}{2}}\right]. \end{aligned}$$

When expanded into a series this becomes

$$I = klV^{\frac{3}{2}}\left[1 \pm \frac{3}{4}\frac{v}{V} + \frac{1}{8}\left(\frac{v}{V}\right)^2 \mp \frac{1}{64}\left(\frac{v}{V}\right)^3 + \cdots\right]. \qquad (3)$$

For the largest heating currents used in the experiments v was only 7 volts so that for values of V from 20 to 200 volts the current will be given with sufficient accuracy by using only the first two terms of the series namely:

$$I = klV^{\frac{3}{2}}\left(1 \pm \frac{3v}{4V}\right). \qquad (4)$$

The Effect of Residual Gases and of Previous Treatment of the Filament on the Current-Voltage Characteristics.

In order to test eq. (4) it was necessary not only to obtain the highest possible vacuum in the apparatus, but also to subject the filament to sufficient preliminary heating to insure a regular behavior of its thermionic emissivity.

The Curves I., II. and III. of Fig. 7 show some of the extreme types of irregularity encountered with a filament that is not sufficiently aged. This filament had on a previous occasion been reduced to the state where its emission was regular and eq. (4) was verified. Before taking the data for the curves of Fig. 7 it had, however, remained cold for two days in a vacuum varying from 10^{-4} to 10^{-3} mm. The temperature of the filament for all of these curves was 2340° K. Curve I. was obtained for increasing voltage and Curve II. for decreasing voltage. The pressure measured on the McLeod gauge was 4×10^{-5} mm. This lack of coincidence between the curves for increasing and decreasing voltages was

observed a number of times, when the pressure in the apparatus was between 10^{-4} mm. and 4×10^{-5} mm.

Sometimes the gauge would record a rise in the pressure when the high voltages were reached and then again even at the high voltage the pressure indicated by the gauge would fall, as though the large electron discharge helped to remove the gas from the apparatus.

The characteristic space charge curve which was finally obtained at a pressure of 5×10^{-5} mm. is shown in Curve IV. Curve III. is seen to coincide with IV. up to about 30 volts, when it suffers discontinuities which must be attributed to the occurrence of collision ionization, for they result in a large and sudden increase in the current. This current reaches a definite saturation value at 80. volts.

The lack of coincidence of the curves for increasing and decreasing voltage is of course due to a rapid time decay of the current at the high voltage. This decay of the current is probably due to the liberation of gas from the walls of the anode by the electron bombardment to which they are subjected. The gas thus liberated reaches the filament probably by first becoming ionized by collision and then being driven to the filament by the electric field. At the surface of the filament this gas either forms a compound with the molybdenum or more probably forms a dense layer close to the surface thus hindering the escape of the electrons from the metal. Gas thus liberated and then adsorbed by the filament or perhaps by the cold parts of the apparatus not subjected to electron bombardment would not be recorded on the gauge. Hence these irregularities in the electron current from the filament may well take place without showing any appreciable changes in pressure on the gauge.

A further study of the effect of residual gases on the discharge, although very desirable, was neglected for the main problem of obtaining the true thermionic emission from the molybdenum. We will therefore consider the curves obtained after the phenomena just discussed have been eliminated. Four such curves are shown, one being Curve IV. of Fig. 7, and the others appearing in Fig. 8.

If these curves are in accord with eq. 4 then the thermionic current plotted as a function of $V^{3/2}(1 + (3v/4V))$ should give a straight line, whose slope when squared and multiplied by a constant involving the radius of the anode and the length of the wire should give e/m the ratio of the charge to the mass of the electron. Thus from eq. 4 if I and V are in amperes and volts respectively the expression for e/m in e.m.u. is:

$$\frac{e}{m} = 8.197 \frac{r^2}{l^2} S^2 \times 10^{16},$$

where S is the slope of the curve.

Substituting the measured values of

$$r = 1.12 \text{ cm.},$$

$$l = 8.45 \text{ cm.},$$

we get

$$\frac{e}{m} = 1.44 \times 10^{15} S^2 \text{e.m.u. gm}^{-1}. \tag{5}$$

There are two ways of finding v, the potential difference along the filament. One is the direct measurement by means of a voltmeter connected across the terminals of the filament. The other and more satisfactory way, since it does not include the voltage drop along that portion of the filament under the guard rings, is to find the intercept of the current-voltage curves with the voltage axis. If this intercept be V_0 then

$$v = \pm \tfrac{4}{3} V_0.$$

The error involved in this method of determining v will not seriously affect the results, since v is itself only a small correction term.

Curve V., Fig. 7, shows I as a function of $V^{3/2}(1 + (3v/4V))$, the values

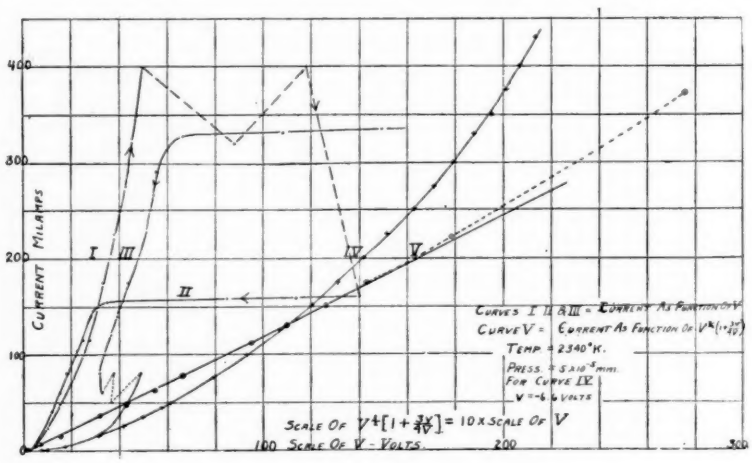

Fig. 7.

of I and V being obtained from the experimental current-voltage Curve IV. Curve V. is linear up to 140 volts at which voltage it begins to curve upward due to the occurrence of sufficient ionization by collision to partially neutralize the space charge. The slope of the linear part of the curve is 1.18×10^{-4} amp. per volt and when substituted in eq. (5) gives

$$\frac{e}{m} = 1.80 \times 10^{7}.$$

Curve II., Fig. 8, showing I as a function of $V^{3/2}$, is plotted from the experimental Curve I. of Fig. 8. It is accurately linear up to 280 volts and to a total current of .5 ampere or about .06 amp. per cm. length of filament. The slope is 1.113×10^{-4} amp. per volt which gives by eq. (5)

$$\frac{e}{m} = 1.782 \times 10^{7}.$$

This value of e/m is within 2 per cent. of the generally accepted value 1.77×10^{-7}.

Curve III. of Fig. 8 conforms to the 3/2 power law up to only 80 volts as shown by Curve IV., in which I is plotted against $V^{3/2}(1 + (3v/4V))$. The departure from the theoretical law is without doubt again due to

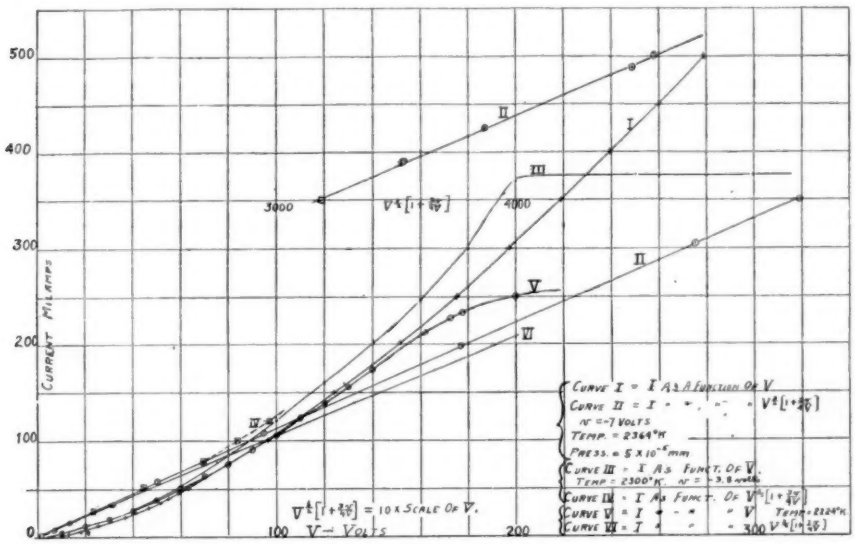

Fig. 8.

the formation of positive gas ions. Other experiments showed that at the temperature of 2300° K., at which this curve was obtained, the saturation current should be .55 ampere. The low value given by the curve is probably due to the action of the gas on the surface of the filament as previously mentioned. The linear part of Curve IV. has a slope of 1.10×10^{-4} and gives

$$\frac{e}{m} = 1.73 \times 10^{7}.$$

Curve V., Fig. 8, conforms to the theoretical current voltage law up to 140 volts, as is shown by the fact that Curve VI. is linear up to this voltage. The current approaches saturation from 150 to 200 volts.

The data for this curve were obtained with a different stock of molybdenum from that used to obtain the other curves. The temperature was 2224° K. Shortly after the conclusion of the experiment giving these data the filament broke near the center, due to excessive tension of the supporting spring. Measurements of the diameter of the wire showed that the central portion was slightly constricted for a length of about 3 cm. This length was therefore probably somewhat hotter than the remaining 5.45 cm. which were also under the influence of the field of the central anode cylinder. This accounts for the smaller value of the slope of Curve VI. and hence for the low value of e/m namely, 1.59×10^7.

A potential of 550 volts was necessary to completely saturate the currents at the temperature of 2364° K., the saturation value of the current being 1.05 amperes. As shown by the curves the space charge equation ceases to hold at 280 volts, and .5 ampere; the current rising more rapidly than required by the equation. This was undoubtedly due to the formation of a sufficient number of positive ions to partially neutralize the space charge due to the electrons. So long, however, as this collision ionization does not become so great as to produce a blue glow in the tube, the saturation current is not appreciably affected. This is shown by the fact that the large saturation current of 1.05 amperes falls accurately upon the saturation current-temperature curves determined at lower temperatures where the pressure and the electron current were so small that practically no positive ions would be formed.

When one considers that the total electron current flowing from the wire to the central anode and the guard rings was 1.8 amperes at 2364° K., it is apparent that to prevent the liberation of absorbed gases from the walls of these electrodes is very difficult. This current at the impressed potential of 550 volts meant that there was just about one kilowatt of energy expended on the walls of the anode and guard rings by the intense electron bombardment to which they were subjected.

Having established that it is possible to obtain in this apparatus a pure electron current or one whose saturation value is not appreciably affected by positive ionization, we will now return to the study of these saturation currents as a function of the temperature.

THE RELATION BETWEEN THERMIONIC CURRENT AND TEMPERATURE IN A HIGH VACUUM.

In the first part of this paper experiments were described which showed that Richardson's equation held approximately when the pressure in the apparatus varied from .0004 to 0.02 mm. of mercury. The con-

stants under these conditions were

$$A = 5. \times 10^{11},$$

$$b = 8.1 \times 10^{4}.$$

Further experiments showed that if the pressure could be held below .001 mm., at a temperature above 1900° C. a change in the thermionic emissivity took place with new values of the constants, namely,

$$A = 10^{8},$$

$$b = 5.5 \times 10^{4}.$$

As to whether these last constants maintained their value when the temperature was again lowered was not determined because of accidents to the apparatus at the high temperature.

In the water-cooled metal apparatus evacuated with a molecular pump it has just been shown that pressures less than 5×10^{-5} mm. of mercury could be maintained at filament temperatures of 2300° K. Furthermore at these temperatures the electron current was limited by the space

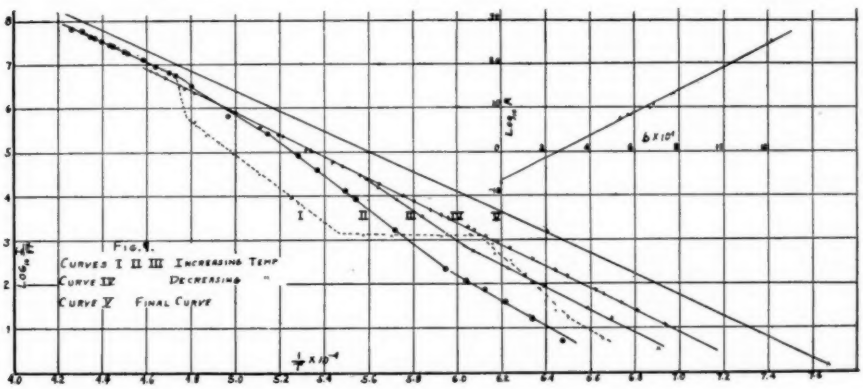

Fig. 9.

charge and varied with the impressed voltage in accordance with the theoretical equation, thus indicating a pure thermionic current; that is a current free from positive ionization. Therefore the vacuum obtained in this apparatus is suitable to determine the thermionic characteristics of molybdenum, that is to determine the true values of the constants A and b of Richardson's equation provided that a pure molybdenum surface can be obtained. To get the constants we will plot the results of the experiments as before, namely $\log_{10} I - \frac{1}{2} \log_{10} T$ as a function of $1/T$. In every case the anode potential was maintained high enough to saturate the thermionic current.

Curve 1, Fig. 9, shows the results obtained from a carefully cleaned

half millimeter molybdenum wire, which after mounting in the apparatus had been heated for two hours at a temperature of 1575° in a vacuum of 2×10^{-5} mm. This preliminary heating was simply to remove the excessive occluded gas from the filament and the surrounding electrodes.

Referring now to the curve we see that up to the temperature of 1625° K. ($1/T = 6.1 \times 10^{-4}$) the thermionic current behaved quite irregularly, and that from this temperature up to 1795° K. it increased only slightly. The behavior during this almost constant current interval was as follows: each increase in temperature was accompanied by an increased thermionic current which fell, however, rather quickly to a constant value. The pressure in the apparatus was 2×10^{-5} mm. while this occurred. From 1800° K. to 2075° K. the curve is linear in accordance with Richardson's equation and gives the constants:

$$A = 1.3 \times 10^{12},$$

$$b = 8.83 \times 10^{4},$$

which agree very nearly with the average values of the constants obtained in the early experiments (see Fig. 2) where the pressure was 100 to 1,000 times as great. The emission of electrons as represented by these constants is not the true electron emission from the metal, nor can it hardly be due to the continued chemical action of gas upon the metal for in this case it would not persist over such wide variations in pressure. Therefore we are led to conclude that it is due to a layer on the surface of the metal which is permanent up to a certain high temperature. Whether this layer is a gas layer held at the surface of the metal by its adsorbing power, or a chemical compound is difficult to determine.

At 2075° K. the emission characteristic of this surface layer has begun to disappear and from 2115° K. up to 2350° K. the curve is again linear with the constants

$$A = 5.4 \times 10^{7},$$

$$b = 5.38 \times 10^{4}.$$

The pressure at these temperatures was 4×10^{-5} mm. These constants agree approximately with the values obtained from the upper curve of Fig. 2 at corresponding temperatures, although the pressure corresponding to Fig. 2 was about ten times as great.

Curve 2, Fig. 9, was taken for ascending temperature after the wire had been exposed to the atmosphere for a short time. Neglecting transition regions, where an insufficient number of points were obtained to determine the slope and where time variations of the current took place, the Curve II. has three distinct values of b and A given in the following table:

Temperature.	b (Deg.^{1}).	A (Amp. Cm.$^{-2}$ Deg.$^{-\frac{1}{2}}$).
1540–1680° K.	6.73×10^{4}	4.66×10^{9}
1680–1950° K.	8.95×10^{4}	2.82×10^{15}
2080–2300° K.	5.38×10^{4}	5.4×10^{7}

The constants obtained at the highest temperature range agree with the values from Curve I. obtained at this same temperature range. The pressure during this run was 2×10^{-5} mm. up to the temperature of 1981° K. when it gradually rose to 1.5×10^{-4} due to the slowing down of the molecular pump by the entrance of oil from the bearings. Before the readings for descending temperatures could be taken the pump behaved so badly that the run had to be discontinued. The pump being run by an induction motor would stop entirely when a little oil leaked in from the bearings.

Curves III. and IV., Fig. 9, were taken for ascending and descending temperatures respectively. Curve IV. continues to the lowest temperature with the same slope that Curve III. assumed at the high temperature, showing very constant conditions on the surface of the hot filament. Curve III. gives the following values of the constants:

Temperature (Deg. K).	b (Deg.^{1}).	A (Amp. Cm.$^{-2}$ Deg.$^{-\frac{1}{2}}$).
1450–1650	5.9×10^{4}	2×10^{8}
1650–1800	7.96×10^{4}	2.80×10^{13}
1800–2170	5.76×10^{4}	2.35×10^{8}

The values at the high temperatures are also those for Curve IV. The pressure throughout this experiment remained below 4×10^{-5} mm.

Curve V., Fig. 9, is reproduced from Curve III., of Fig. 10, and shows the final regular behavior of the current for both rising and falling temperatures.

Curves I. and II., Fig. 10, are for rising and falling temperatures respectively and were taken three days after Curve IV., Fig. 9, during which time the wire remained in a pressure below .005 mm. The pressure during the experiment in which the data for these curves were obtained remained below 7×10^{-5} mm. Curve I. has the following values of Richardson's constants for different temperature ranges:

Temperature (Deg. K).	b.	A.
1500–1720	6.91×10^{4}	3.2×10^{10}
1750–2100	5.88	2×10^{8}
2120–2380	5.90	4.2×10^{8}

Curve II. for decreasing temperature is satisfied throughout its entire course from 2380° K. to 1390° K. by the same constants as Curve I. has at the high temperatures, *i. e.*, from 2120° K. to 2380° K.

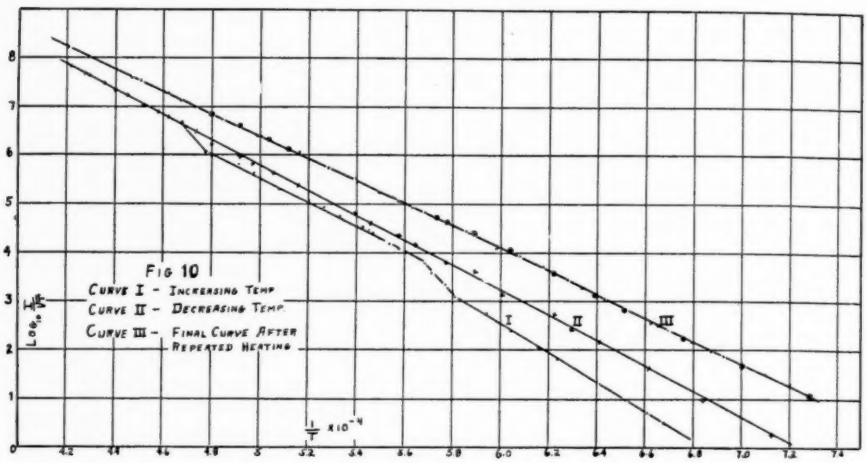

Fig. 10.

The lack of coincidence of the currents obtained for rising temperature and those obtained for falling temperature was due to a time decay of the current at low temperatures and a growth of the current at high temperatures. The curves shown in Fig. 11 show the decay of the

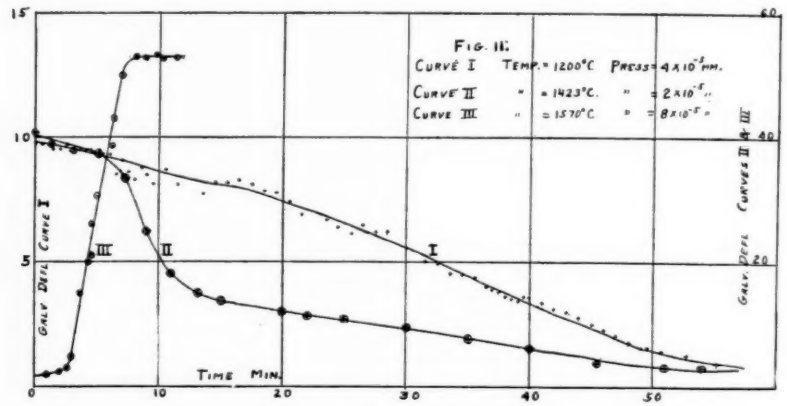
FIG. 11.
CURVE I TEMP. = 1200°C PRESS = 4 X 10⁻⁵ MM.
CURVE II " = 1423°C. " = 2 X 10⁻⁵ "
CURVE III " = 1570°C " = 8 X 10⁻⁵ "
GALV. DEFL. CURVE I
GALV. DEFL. CURVES II & III
TIME MIN.
I
II
III

Fig. 11.

current at temperatures of 1200° C. and 1423° C., and its growth at 1570° C. The curves show that the current variations are rather irregular being at some temperatures approximately exponential and at others almost linear. For a filament which had been repeatedly heated the current showed a much slower rate of decay at the low temperature

than for a newer filament. The decay of the current is probably due to the adsorption of gas by the molybdenum surface and the growth of the current, to the reemission of this gas at the high temperatures. These processes would be influenced by the amount of gas present; and also perhaps by the number of ions formed, which depends upon the density of the electron current from the filament. After repeated runs both the filament and the anodes were more completely freed from gases and hence these effects became less. Once having attained a regular behavior such as represented by Curves II. or III. of Fig. 10 the currents obeyed Richardson's equation for both rising and falling temperatures which shows that no rapid time variations of the current were taking place. If, however, the filament were allowed to stand cold in vacuo for a day or longer the currents obtained for rising temperature would not coincide with those obtained for falling temperature showing that the surface had again become contaminated. After repeated heatings at intervals during the course of about a month the time variations of the current, as well as the current temperature hysteresis effect shown by these earlier curves became less marked, due no doubt to the more complete removal of adsorbed gases from the filament and from the surface of the anode cylinders. Curve IV., Fig. 10, shows the final regular behavior of the thermionic current, and gives the following values of the constants: $A = 1.1 \times 10^8$, $b = 5.36 \times 10^4$.

These constants are not necessarily the ones which might be obtained if it were possible to have a perfectly pure molybdenum surface in a perfect vacuum. They may simply correspond to another stable condition of the surface such as was found at the higher pressure, which condition is produced by continued heating in pressures of about 5×10^{-5} mm. This stable condition persists over a wide range of temperature but if the filament could be studied in a still higher vacuum it might be possible to obtain surface conditions giving still lower values of A and b.

In order to indicate how well the thermionic saturation currents could be reproduced after this final regular behavior of the emission was reached the curves shown in Fig. 12 were plotted. The ordinates give the saturation current in amperes per square centimeter of emitting surface. Each curve after the first is plotted to ten times the scale of ordinates of its predecessor. The results of four different experiments performed at different times on the same molybdenum wire are shown by the variously designated *crosses* on the curves. From 1050° C. to 1750° C. all of these points lie accurately upon the curve which has the constants $A = 1.1 \times 10^8$ and $b = 5.36 \times 10^4$. Above 1750° the currents were not always so accurately reproducible. This was due to the greater pressure in the apparatus at the higher temperatures.

The points on Fig. 12 designated by dots surrounded by circles are results from a different molybdenum wire which came to grief, however, before it was heated quite long enough to give a regular electron emission.

At the temperature of 1950° C. there is also shown in Fig. 12 the phenomenon of so-called temperature saturation with constant applied voltage. As pointed out by Langmuir this effect is due to the disappear-

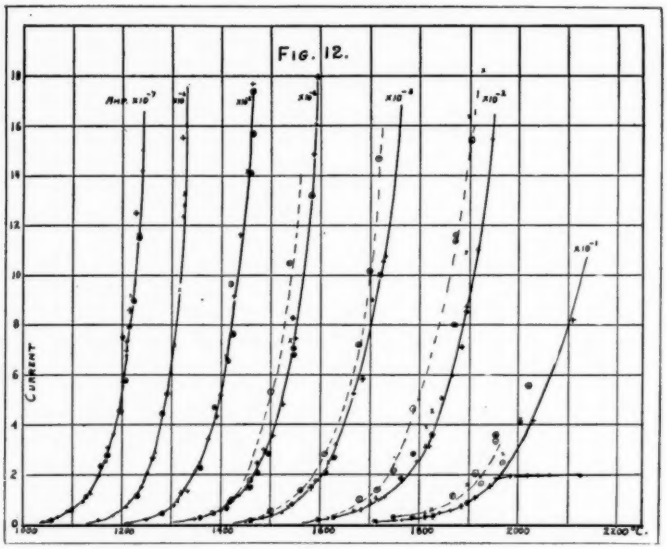

Fig. 12.

ance of the electric field near the wire on account of the accumulation of electrons in the space between the wire and the anode. The potential of the anode with respect to the wire when this occurred was 175 volts, whereas the potential used for saturating the current was 500 volts.

THE RELATION BETWEEN THE CONSTANTS A AND b.

H. A. Wilson[1] has shown that the values of A and b obtained for platinum in various pressures of hydrogen are related in the following manner:

$$\log A = cb + d, \qquad (1)$$

where c and d are constants.

O. W. Richardson[2] in a recent paper collected the various determinations of the constants A and b for tungsten and for platinum and notwithstanding that the constants were determined under a variety of conditions and with different gases present, they approximately obeyed

[1] Phil. Trans. (A), Vol. 208, p. 247 (1908).

[2] Proc. Roy. Soc. (A), Vol. 91, p. 524 (1915).

the relation (1). The following brief abstract of Richardson's theory of the effect of gases in changing the values of the constants A and b is given to make clear its application to the results obtained from molybdenum.

Richardson assumes that in addition to the work b which an electron must do to free itself from the pure metal there is an amount of work φ which is proportional to the change in the contact P.D. caused by the contamination of the surface by the gas. If V be this contact P.D. of the contaminated surface against the pure metal, k the Boltzman constant and e the elementary charge then

$$\varphi = \frac{eV}{k}. \tag{2}$$

The thermionic current from the contaminated surface thus becomes

$$I = AT^{\frac{1}{2}}e^{-\frac{(b+\phi)}{T}}. \tag{3}$$

Further if φ is assumed independent of the kind and pressure of the gas as the experiments with tungsten seem to indicate, it still might depend on the temperature, the simplest assumption being that

$$\varphi = a_0 + a_1 T, \tag{4}$$

where a_0 and a_1 are constants. Hence the current from the contaminated surface is

$$I = Ae^{-a_1}T^{\frac{1}{2}}e^{-\frac{(b+a_0)}{T}}, \tag{5}$$

which may be written

$$I = CT^{\frac{1}{2}}e^{-\frac{d}{T}}.$$

Where C and d denote the general values of A and b for a contaminated surface.

Then $C = Ae^{-a_1}$ or $\log C - \log A = -a_1$ and $d = b + a_0$ or $d - b = a_0$ whence

$$\log C - \log A = \frac{a_1}{a_0}(d - b). \tag{6}$$

Richardson's curves of $\log_{10} A$ as a function of b show that $-(a_1/a_0)$ is a positive constant for both tungsten and platinum. In the experiments on molybdenum described in this paper values of the constant A ranging from 10^{-7} to 10^{25} and corresponding values of b from 0 to 12 were determined. In the upper right-hand corner of Fig. 9 the values of $\log_{10} A$ are shown plotted against the corresponding values of b. The points lie very much more accurately on a straight line than do the diversely determined values collected and plotted by Richardson for

tungsten and platinum. With the exception of the zero value of b the small values of b and A result from the molybdenum surface obtained after long-continued heating in the lowest pressures and therefore correspond to a higher degree of purity of the surface. Therefore it follows that in eq. (6) both $\log C - \log A$ and $d - b$ are positive so that $-(a_1/a_0)$ is also positive for molybdenum. Its value determined from the slope of the curve is 5.86×10^{-4}.

Replacing $-(a_1/a_0)$ by α in equation 4 we have

$$\varphi = a_0(1 - \alpha T)$$

and from equation 2 we can calculate the contact P.D. (V) of contaminated molybdenum against pure molybdenum, viz:

$$V = -\frac{k}{e} a_0(1 - \alpha T) \times 300 \text{ volts.}$$

Assuming 5.3×10^4 to be the value of b for the pure molybdenum surface we can calculate the contact P.D. of the most contaminated surface corresponding to $b = 12.1 \times 10^4$ at the temperature of say 1640° K. beyond which temperature this surface did not persist in a good vacuum ($p < 10^{-4}$). This calculation gives

$$V = -0.224 \text{ volt.}$$

In the earlier experiments where the pressure was greater than 10^{-3} mm. it was found that a stable condition of the surface existed and gave the values

$$A = 5 \times 10^{11}, \qquad b = 8.1 \times 10^4.$$

Again assuming that $b = 5.3 \times 10^4$ for the pure metal the contact P.D. of the contaminated surface necessary to give the above constants at 2000° K. would be $+ .42$ volt.

Richardson also suggests that this contact P.D. may be due to the presence of positive or negative gas ions in the surface layer on the metal. This suggestion would agree with the suggested explanation of the time fatigue of the current at high voltages and its recovery at low voltages. For the number of such ions formed in the space, and adsorbed by the surface per unit of time would increase with the potential thus causing the fatigue at high voltages. At the lower voltages these gas ions would again be driven from the surface at a greater rate than they would be adsorbed by it, thus causing the recovery of the current.

This suggestion might further be applied to explain the zero value of b obtained in one of the experiments (see Curve I.), Fig. 9. As previously mentioned the thermionic current during this part of the experiment would rise with each increase in the temperature and then

rapidly decay to a value corresponding to $d = 0$. This decay could be explained by the formation of a surface layer which continued until a_0 became equal to $-b$ so that d, the value of the constant b, for the contaminated surface would become zero.

SUMMARY.

1. At pressures above .001 mm. the saturation thermionic currents from molybdenum obey Richardson's equation approximately with the following values of the constants

$$A = 5. \times 10^{11},$$
$$b = 8.1 \times 10^{4}.$$

The current at these pressures is not, however, a pure electron current as shown by its failure to satisfy the theoretical equation between the current and anode potential.

2. A visible coating of molybdenum oxide increases the electron emission from the metal, above that of an apparently clean surface. This oxide coating volatilizes or decomposes at about 1400° C. and with its disappearance the electron emission decreases.

3. In a pressure of the order of 5×10^{-5} mm. the electron currents from a hot molybdenum wire cathode to a concentric cylindric anode are shown to satisfy the theoretical relation between the current and voltage up to 280 volts and .06 ampere per cm. length of the wire. Four values of the ratio e/m are calculated from the curves and agree well with the generally accepted value. Before this pure electron emission is obtained the filament must be heated and the anode subjected to electron bombardment a number of times. During this treatment it was found that the thermionic current decayed at the high voltages and recovered at the lower voltages. This effect is attributed to the liberation of gases from the anode by the more intense electron bombardment to which it is subjected at the high voltage, and also to the ionization of the gases thus liberated and their consequent accumulation near the surface of the hot cathode. When the voltage is lowered these gases are driven from the filament because of its high temperature and so the electron emission increases. Because of the observed increase of the current by a film of oxide, it is not thought that these gases form a chemical compound on the surface of the filament, but rather that they create an electrical potential which hinders the escape of the electrons.

4. A study of the saturation currents as a function of the temperature in vacua of the order of 5×10^{-5} mm. showed the following:

The initial behavior of a perfectly clean molybdenum wire is very

irregular even at these low pressures. It is in general characterized by larger values of the constants A and b than the values given in its final condition. Above 2100° K. the behavior of the emission conforms to Richardson's equation with the following values for the constants:

$$A = 1.1 \times 10^8,$$

$$b = 5.36 \times 10^4.$$

This characteristic emission persists when the temperature is again lowered; but if the filament is kept at a low temperature for a sufficient time its emission decays down to that represented by the larger values of b and A. Since an oxide on the surface increases the emission, this decay is attributed to the adsorption of gas by the filament at the low temperature rather than to the formation of a chemical compound.

5. The numerous values of the constants A and b corresponding to different conditions of the molybdenum surface were plotted and showed that $\log A$ is a linear function of b. This is in accord with the results of H. A. Wilson and O. W. Richardson on platinum and tungsten. Richardson's theory of the effect of gases on the constants A and b is applied to determine the contact potential of the most contaminated molybdenum surface against the pure metal. This is found to be -0.224 volt at 1640° K.

The contact potential of the stable surface condition corresponding to the values of A and b in the early experiments at relatively high pressures is also determined and found to be 0.42 volt at 2000° K.

In concluding I wish to express my thanks to Professor C. E. Mendenhall, under whose direction the above work was performed.

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IONIZATION BY IMPACT IN MERCURY VAPOR AND OTHER GASES.

I. THE DIRECT MEASUREMENT OF THE IONIZING POTENTIALS.

BY F. S. GOUCHER.

Introduction.—The purpose of this part of the investigation was to introduce a modification into the method for the direct measurement of the ionizing potentials of different gases, with the end in view of eliminating certain sources of error which have been present in all the applications of the method yet made.

The principle of the method we owe to Lenard;[1] it consisted essentially in the acceleration of free electrons through an electric field whose strength was varied until sufficient energy had been imparted to the electrons to produce ionization in an ionization chamber. Whereas Lenard and also Dember[2] used photo-electric electrons as the means of producing the ions, those emitted from a hot filament have been used by v. Baeyer,[3] Franck and Hertz,[4] and Pawlow.[5] In all cases, however, the exact point at which ionization set in was rendered doubtful because of a lack of homogeneity of velocity among the electrons. This lack of homogeneity arose in consequence of two causes in the cases where the hot filament was used, viz., the *IR* drop of potential along the wire, and the initial velocity of the electrons themselves due to the high temperature of the filament.

Measurements carried out by Franck and Hertz and also by Pawlow for the determination of the actual distribution of velocity among the electrons when they were accelerated through a definite applied field, revealed the fact that there was always a considerable number having a velocity of one volt or more greater than the velocity corresponding to the field applied. This amounted to 5 per cent. of the number having a velocity corresponding to the applied field, in the case of the measurements made by Franck and Hertz and considerably more in those of Pawlow.

[1] P. Lenard, Ann. d. Phys. (4), 8, 149, 1902.
[2] H. Dember, Ann. d. Phys. (4), 30, 137, 1909.
[3] O. v. Baeyer, Verh. d. D. Phys. Ges., 10, 96, 1908.
[4] Franck and Hertz, Deutsch Phys. Ges., Vol. 15, p. 34, 1913.
[5] Pawlow, Proc. Roy. Soc., Vol. 90, p. 390, 1914.

Although in both of these cases the majority of the electrons had velocities corresponding to the voltages applied and the effect of the small number having excess velocity could not have been large, yet if a knowledge of the absolute value of the ionizing potentials of the different gases measured is desired this lack of homogeneity is sufficient to render the value uncertain, as was pointed out by Franck and Hertz. The criterion for the amount of correction necessary in consequence of this is the constancy of the corrected value under different experimental conditions of temperature of the filament and pressure of the gas.

It therefore seemed desirable to minimize these two sources of error: and it was with this purpose in view that this part of the experiment was undertaken. The modification introduced was that of obtaining the electrons from an equipotential hot surface, of sufficiently large area to furnish at relatively low temperatures enough electrons for the measurement.

The method, thus modified, was to be applied to the determination of the ionizing potentials of several gases and in particular mercury vapor.

A direct determination of the value for mercury vapor seemed desirable because the values already obtained by Franck and Hertz[1] and Newman[2] have been recently called in question in a paper by J. C. McLennan,[3] on the ground that if his interpretation of the Bohr hypothesis be correct, then the interpretation put on their experiments by Franck and Hertz and Newman could not be correct.

Franck and Hertz showed that impacting electrons lost their energy at $4.9 \pm .1$ volts, whereas below that value the impacts were elastic, they therefore considered this equivalent to ionization and that 4.9 was the value of the ionizing potential in mercury vapor. It was also shown by them[4] that under these conditions—the impacting electrons having an energy corresponding at least to 4.9 volts—the single line spectrum $\lambda = 2536.7$ Å.U. was emitted. Further that the value of voltage yielded by the quantum relation $Ve = h\nu$, when ν had the frequency corresponding to this wave-length, was exactly 4.9 volts.

McLennan and Henderson[5] extended this work and showed that for the elements Hg, Cd, Zn, and Mg, a single line spectrum was produced, by the application of a certain minimum energy, and further that at a certain value of energy greater than this the many-lined spectrum set in. This latter, in the case of Hg vapor corresponded roughly to the value

[1] Franck and Hertz, Verh. d. D. Phys. Ges., 16, S. 457, 1914.
[2] Newman, Phil. Mag., Vol. 28, pp. 753–756, Nov., 1914.
[3] McLennan, Proc. Roy. Soc., *A*, Vol. 92, 305, 1916.
[4] Franck and Hertz, Verh. d. D. Phys. Ges., Vol. 11, p. 512.
[5] McLennan and Henderson, Proc. Roy. Soc., *A*, Vol. 91, 485, 1915.

calculated from the quantum relation when the value ν was taken to correspond to the limiting line, $\lambda = 1188.0$ Å.U. of the Paschen series, $\nu = 1.5$, S − m, P.; whereas the line $\lambda = 2536.7$ Å.U. corresponded to the first member of the Paschen series $\nu = 2$, ρ − m, S. It was argued by McLennan that in the light of the Bohr hypothesis ionization could not be said to have taken place until sufficient energy had been applied to cause the atom to emit its many-lined spectrum including the head of the series; in which case the value 4.9 volts could not be correct.

Although the method used by Franck and Hertz did not show directly whether negative electrons had been set free from the atom or not, the direct method does; and it therefore seemed advisable to make a careful measurement by the direct method as modified in this experiment.

Apparatus.—The essential feature of the modification introduced into the method was the equipotential surface electron source,[1] a sectional view of which is shown in Fig. 1. A platinum thimble T − 7.6 cm. long, .95 cm. in diameter and .01 cm. thick—was heated by a tungsten spiral H, suspended within it. T was sealed into the glass part G, at V; K being merely a support for other parts of apparatus.

The heater required about 30 watts' power, and heated the complete upper half of the thimble to a uniform temperature. The leads of the heater consisted of a heavy copper wire R and a thin copper sheath C insulated from R by a glass capillary tube I. One end of the tungsten spiral was attached to R and the other to a loop in the upper end of a tungsten wire support, S, which in turn was fastened to C. The wire support fitted the thimble T closely touching it at two points only, but these were corresponding points in a divided circuit so that no potential difference existed between them. Consequently T could be maintained at any desired potential by means of the lead wire W in metallic contact with it; the current used in heater being furnished by an insulated storage battery. M was a mica cylinder surrounding C, insulating it from T and serving to keep it concentric with T. Further, it is evident that the sheath C was practically at the same potential as T and prevented the field within it due to the IR drop along the spiral from having any effect on the electrons given off from T.

Fig. 2 shows the complete assemblage of parts as it was used for making the measurement in Hg vapor. The essential features are the electron source T; a fine platinum gauze G completely surrounding it; and a collecting electrode E of thin sheet aluminum supported and insulated by a quartz tube sealed into the side arm of the enclosing vessel.

[1] The suggestion of the use of a separate heater was given to me by Prof. J. H. Morecroft, of the Department of Electrical Engineering.

All joints S were of De Khotinsky cement excepting G' which was glass.

The parts were so designed that these joints were at some distance from the main body of the vessel, on account of the heat from the heater itself, and because the measuring chamber had to be heated to a temperature of 60° to 70° in order to obtain sufficient mercury vapor density.

Fig. 1.

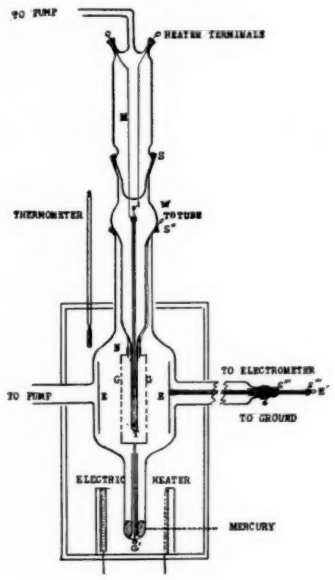

Fig. 2.

Also it is seen that the glass parts holding the thimble T and supporting the heater terminals could be separately removed, for convenience in replacing heater, lime coating cathode, and inserting liquid mercury in vessel.

The relative dimensions of the apparatus can readily be estimated from the drawing; given that the distance from T to G is about .8 cm., and that from G to E 2.5 cm. The supporting electrode EE' was about 30 cm. in length.

The double wall represents an asbestos lined box entirely enclosing the vessel and containing an electric heater and supporting a thermometer.

The two exhaust tubes of the vessel were connected through separate stopcocks to a Gaede molecular pump, and a McLeod gauge. The tube from the main vessel contained a large pentoxide drying tube and a freezing out tube.

The electrical measurements were made by a suitably shielded electrometer of the Dolezalek type, and sensitive to about 1,000 div. per volt,

used in connection with a variable air condenser and Wolff potentiometer. The potentials were applied by means of dry cells and suitable potentiometer connections, and were measured by a Siemens and Halske standard voltmeter.

I would here like to express my thanks to the Research Laboratory of the General Electric Company which so kindly furnished the tungsten used in this apparatus.

Method.—The method of measurement was essentially that employed by Franck and Hertz and Pawlow.

With suitable conditions of gas pressure within the vessel, the electrons were accelerated through the gauze, under different applied fields, until positive ions were formed in the region beyond. The fields were so arranged that the positive ions were drawn to the collecting electrode, their rate of formation being measured. This positive ion current was determined for a series of values of accelerating field and plotted in the form of a curve, the point where this current curve cut the voltage axis giving the value of voltage at which positive ionization set in under these conditions.

The determination of the distribution of velocities among the electrons for a given accelerating voltage was obtained by measuring the negative electron current as a function of an opposing field applied between the gauze and the collecting electrode, and plotting in the form of a curve. This curve was graphically differentiated and a new curve plotted, giving the relative number of electrons having velocities corresponding to the different opposing fields. The pressures used were, of course, very low and the accelerating voltage was always less than the ionizing voltages for the gas present.

Experimental Results.—The gain in homogeneity of velocity among the electrons accelerated by a given applied field, from the equipotential surface electron source, over that obtained from the hot wire is best shown by a comparison of the velocity distribution curves obtained under similar conditions.

Fig. 3 shows such a comparison. Curve (*a*) represents the velocity distribution among the electrons from the equipotential surface source when accelerated by a field of 7.5 volts, in hydrogen below .005 mm. pressure; the total electron current being taken as 100 per cent. Curve (*b*) shows the distribution obtained by Pawlow under like conditions.

It is seen that at the pressures used over 70 per cent. have velocities corresponding to the applied voltage in the case of (*a*) whereas less than 35 per cent. have the corresponding velocity in the case of (*b*); further, the number having a velocity corresponding to .5 volt greater than the

applied field in the first case was too small to be measured, whereas a measurable amount have velocities corresponding to at least 2 volts more than the applied field in the second case.

From this it can be concluded that a very high percentage of all the electrons emitted from the equipotential surface electron source have practically a zero velocity of emission; and that consequently initial

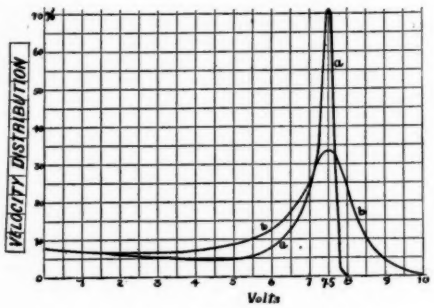

Fig. 3.

conditions approximate more closely those necessary for the determination of the absolute value of the ionizing potential than they do with a hot wire as a source.

The positive ionization curves obtained under such initial conditions of electron source show that a considerable variation of the temperature of the same will produce no change in the break point although increasing the temperature unduly will cause ionization to set in at lower voltages as would be expected. This indicates that under these conditions no correction need be made for the effect of those few electrons having initial velocities greater than zero. Similarly the curves show that changes of pressure do not affect the break point.

Fig. 4 shows two curves obtained with hydrogen present in the tube, the pressure being the same for each at .01 mm. Hg and the temperature of the electron source being varied T_2 greater than T_1. Fig. 5 shows two curves taken under the same conditions of electron source but different conditions of pressure.

The agreement and invariability of the break point at about 10.25 volts would indicate that this value corresponded closely with the absolute value of the ionizing potential for this gas, although Franck and Hertz and also Pawlow found 11 volts for hydrogen. That this was due to the pure hydrogen was considered at first to be highly probable since the hydrogen was obtained electrolytically and the constancy of pressure showed that the amount of gas given off from the hot source was extremely small. That mercury vapor had any effect was con-

sidered unlikely since no mercury had ever been in the vessel when these measurements were made and the freezing tube between the vessel and the McLeod gauge was kept immersed in an ice and $CaCl_2$ mixture. Further, it was assumed that since the ionizing potential of mercury vapor had been found to be 4.9 volts, its effect, if present in sufficient

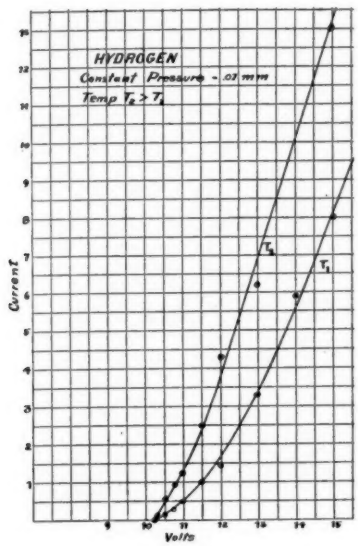

Fig. 4.

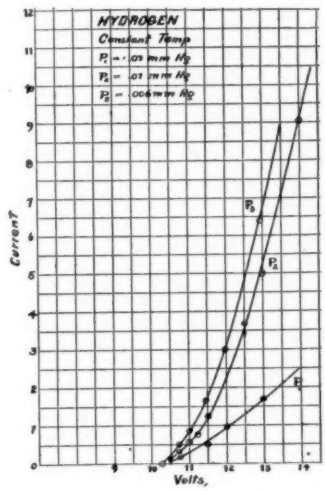

Fig. 5.

quantity, would have taken place below 10 volts. However, in the light of later results as will be pointed out, it does not seem unlikely that the slight amount of mercury vapor present, even though considerably below .0001 mm. pressure, could have been responsible for this value $10.25 \pm .1$ volts.

Nitrogen.—Fig. 6 shows a similar set of curves for nitrogen obtained under conditions of constant pressure, viz: 0.01 mm. Hg, but with a different temperature of the source for each curve, T_3 being greater than T_2 and T_2 being greater than T_1. Here again as in the case of hydrogen the break point is the same for the three curves within a small range; and this occurs at about 7.4 volts, which is very close to the value obtained for nitrogen by Franck and Hertz, viz: 7.5 volts. The nitrogen used was made by the department of chemistry according to their most refined methods and was considered by them to be quite pure.

Mercury Vapor.—Fig. 7 shows the set of curves obtained in the case of mercury vapor, in which curves $T_1T_2T_3$ were obtained with different temperatures of the electron source, T_3 being at the highest temperature. The velocity distribution curve shown in Fig. 8 was obtained with the

source at the temperature corresponding to that used when the curve T_2 was obtained.

For measurements obtained in mercury vapor the platinum thimble

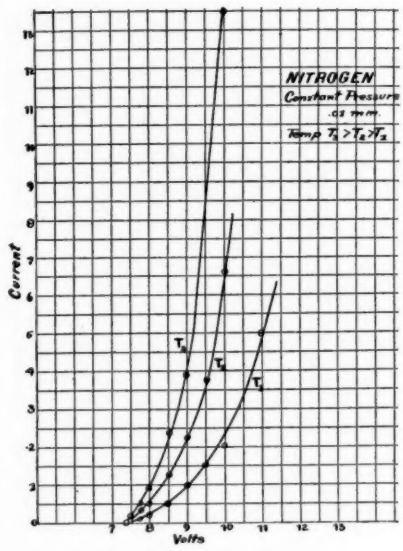

Fig. 6.

was coated with CaO and heated for a long time to free it of gases—mercury having been inserted into the tube as is shown in Fig. 2. The temperature of the whole vessel inside of the asbestos-lined box was brought up to about 55° for an hour or so before the measurements were made. Also the molecular pump was kept running to carry away traces of gas given off from the source and the walls of the vessel.

It is to be noted that the curves thus obtained are very steep, making very abrupt breaks at the voltage axis. This is what we should expect in the light of the fact that the impacts between electrons and molecules of mercury vapor have been shown to be elastic, when the electrons have an energy less than that necessary to produce ionization. For under this condition practically all the electrons given off by the source will be available to produce ionization as soon as a sufficient field has been applied, which is not the case when the impacts take place in the more non-elastic gases as hydrogen and nitrogen.

The break points in this case take place in a region between 4.8 and 4.9 volts which is in good agreement with the value 4.9 volts obtained by Franck and Hertz by their method and the value given by calculation from the quantum relation $Ve = h\nu$, where ν is the frequency of the

line spectrum emitted $\lambda = 2536.7$ Å.U., as pointed out by Franck and Hertz.

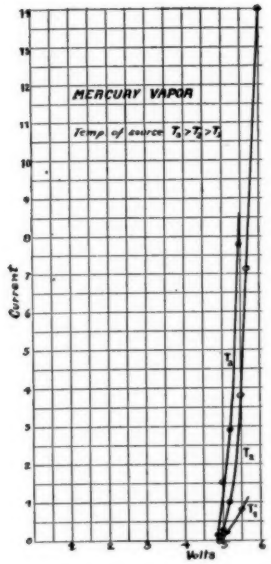

Fig. 7.

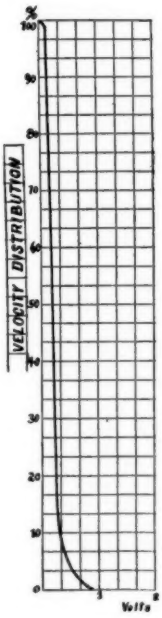

Fig. 8.

This furnishes then, additional evidence in support of this value as the true ionization potential for mercury vapor.

II. The Form of the Ionization Curve over a Range Greater than Twice the Ionizing Voltage.

Introduction.—The purpose of the second part of this investigation was to secure experimental data on the form of the current curves over a range greater than $2V_0$, where V_0 is the ionizing potential, both in the cases of elastic and non-elastic gases.

The theoretical form of the current curve for the case of a non-elastic gas has been given by Bergen Davis,[1] for a range of voltages extending from V_0 to $3V_0$.

Although the theoretical form was not worked out for the case of elastic gases, it was thought advisable to obtain data on the curve shape for an elastic gas under similar conditions to those obtaining in the case of the non-elastic gas.

Apparatus.—As the theoretical form of the curve given by Prof. Davis was expressed in terms of applied voltages, dimensions of the

[1] Bergen Davis, PHYS. REV., N. S., Vol. V., Feb., 1915.

apparatus, and pressure of the gas; it was convenient to make the dimensions of the apparatus to conform to the dimensions assumed for the calculation of a specific case as given in his paper.

To do this it was necessary to modify the form of measuring vessel to that shown in Fig. 9. The cylindrical form of electrodes could not be used, since the assumptions made in the theory were that the electrons were emitted from a plane surface with no initial velocity of their own; that they were accelerated through a gauze parallel to the electron source; and that the positive ions formed were collected by a collecting electrode also parallel to the source. The distances in the specific case calculated were taken as 1 cm. between source and gauze and 1 cm. between gauze and collecting electrode.

The vessel therefore was made to fulfill these conditions. The platinum thimble electrode T was used as before excepting that in this case only the upper end of the thimble was heated, and a guard ring P of platinum foil was supported in the plane of the end by means of a thin aluminum cylinder supported by the glass part K. (of Fig. 1). This guard ring was in electrical contact with T. The platinum gauze G and the collecting plate electrode E were each parallel to the plane of the guard ring P; G being 1 cm. distant from both T and E. In other respects the vessel was similar to that used in the first part of the experiment.

EXPERIMENTAL RESULTS.

(*a*) *Case of Non-elastic Impacts.*—The gas used in this case was hydrogen, because of the possibility of maintaining its pressure very uniform throughout a series of measurements. This was done by means of a stream method; the hydrogen was allowed to pass through a palladium tube—heated by an electrical heater—at such a rate that, with the pump running continuously, the pressure did not vary over quite a considerable period of time. The pressure was chosen so that the value of the mean free path of the electron for this pressure was the same as the value postulated in the calculation of the specific case by Davis. In this way the conditions were, as nearly as possible, made to comply with all the assumptions postulated in the case calculated.

It was found, however, that the number of electrons getting through the gauze at very low pressures varied with the voltages so that the gauze absorbed electrons, consequently the curves obtained under these conditions had to be corrected to correspond to the conditions of a non-absorbing gauze. This was done by getting the percentage change of total electron current getting through the gauze for the range of voltages used and correcting the curve shape accordingly.

The pressure used to fulfill this condition was about .03 mm. Hg. The form of the corrected curve obtained is given in Fig. 10, curve (*a*), curve (*b*) being the theoretical curve. It is seen that the agreement is

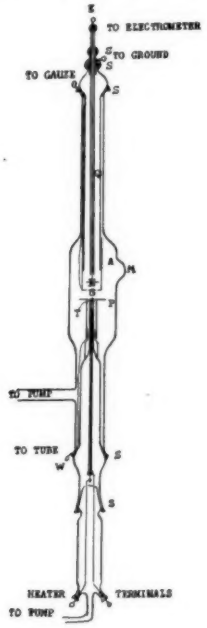

Fig. 9.

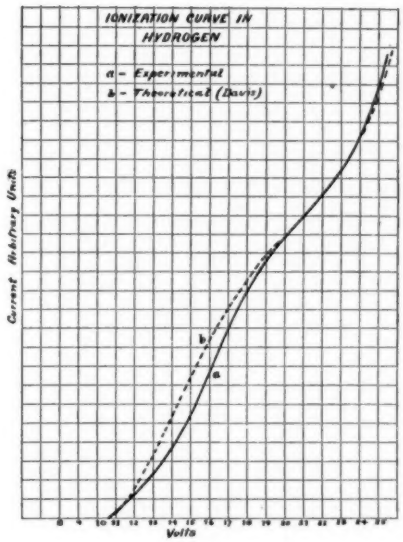

Fig. 10.

very good, and that the general form is well predicted by the theory, furnishing support for the assumptions made.

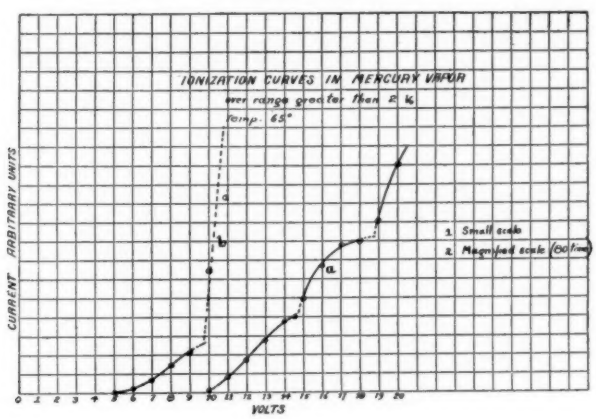

Fig. 11.

(*b*) *Elastic Impact.*—For this case the measurements were made in mercury vapor. Mercury was inserted into the tube which was sup-

ported in a horizontal position, so that the mercury formed a pool at M, and the whole was heated in an asbestos-lined box as was done in the case of the determination of the break points in mercury vapor. At first it was impossible to detect any positive ions until about 10 volts had been applied between the electron source and the gauze, when a very strong ionization set in, but it was found that by increasing the sensitivity and heating the source hotter some ionization took place between 5 and 10 volts. Thinking that the effect at 10 volts might be due to hydrogen emitted by the platinum thimble, an electrode supporting a single tungsten filament about 1 cm. in length and conducting suitable leads, was substituted for the equipotential surface electrode. This was heated for an hour or so at a very high temperature so that the residuum of gas must have been very small,—the pressure as shown by the McLeod gauge showing no variation when the filament was heated to the temperatures used in making the measurements. With this as the electron source the curves Fig. 11 were obtained, curve (b) being on a very much larger scale than curve (a). It is seen that the niches in (a) occur at approximately 3 and 4 times V_0, when V_0 is taken as slightly under 5 volts. The dotted lines signify that at these places the form of the curve is uncertain. The experimental value lying on curve (b) at the value 10 volts was about 80 times the corresponding value lying on curve (a), so that if curve (a) were plotted on the same scale as (b) it would assume a position somewhat near that shown by the dotted line.

Although from these data alone no quantitative conclusions can be drawn as to the relative magnitude of the ionization taking place at 5 and in the region of 10 volts, it however, would appear that there probably are two distinct types of ionization, as McLennan pointed out might be the case. The value at which the sharp rise occurs is in good agreement with the value 10.27 volts calculated by him from the quantum relation when the frequency was taken as the value corresponding to the head of the series spectra emitted.

It has already been shown that negative electrons are set free at 4.9 volts. Whether the marked increase in positive current be due to a greatly increased number of positive ions from which only one electron has been freed, or whether the positive charge on each ion formed has been increased due to the freeing of many electrons, it is, of course, at present impossible to say. But further experiments are in progress on this point.

It is now evident why the value $10.25 \pm .1$ volts as found for hydrogen may be due to this second type of ionization in mercury vapor, especially since this value agrees so well with the calculated value.

On account of the large quantities of mercury present in the vessel and on the electrodes as a result of the work in mercury, measurements for hydrogen, free of mercury vapor, could not be conveniently again undertaken without new apparatus. The value 11 volts obtained by Franck and Hertz and Pawlow, however, is probably correct, as they used liquid air. It is evident that in the case of the curve Fig. 9 the curve shape could be very little changed as the values 11 and 10.2 are so near together, on this scale, and further the per cent. of mercury vapor was considerably smaller in this case, due to the larger pressure of hydrogen used.

I.

Summary.—(*a*) A modification of the direct method for the determination of the ionizing potentials of different gases has been developed. As a source of electrons a platinum thimble heated from within by an auxiliary heater was employed, the electron emission thereby taking place from an equipotential surface, of comparatively large area, so that lower temperatures can be used than with heated wires. In this way the homogeneity of velocity among the electrons under a given applied field was greatly increased.

(*b*) This method when applied, gave for hydrogen $10.25 \pm .1$ volts nitrogen $7.4 \pm .1$ volts, mercury vapor $4.9 \pm .1$ volts, which with the possible exception of hydrogen are in good agreement with the values as obtained by Franck and Hertz.

II.

(*a*) By specially arranged apparatus the form of the current curve for the non-elastic gas hydrogen was obtained, extending over a range greater than $2V_0$, and found to check well the theoretical form of such curves as developed by Bergen Davis, thereby lending support to his assumptions.

(*b*) The form of the elastic gas current curve over a range equal to $4V_0$ was obtained for mercury vapor under similar conditions to those holding in the case of (*a*): the form of this indicates the probability of a second type of ionization in the region of 10 volts and that this value corresponds to that at which McLennan and Henderson have shown the many lined spectrum to be formed.

In conclusion I would like to express my gratitude to Prof. Davis, under whose supervision the experiments were carried out, and to Prof. Pupin for the facilities of the Marcellus Hartley Research Laboratory, and for his interest in the work.

Columbia University,
July, 1916.

THE MAGNETIC SPECTRA OF THE β-RAYS OF RADIUM DE AND OF RADIUM AND ITS PRODUCTS, DETERMINED BY THE STATISTICAL METHOD.

BY ALOIS F. KOVÁŘÍK AND L. W. MCKEEHAN.

INTRODUCTION.

THE investigation of the magnetic spectrum of the β-rays of radium DE was undertaken as a preliminary to the study of the ionization of a single β-particle and the dependence of this ionization upon velocity.[1] The band in the magnetic spectrum of radium E seemed well adapted to such an investigation. The spectrum of the β-rays from radium and its products was also obtained although in less detail. The results in the latter case differ materially from those obtained by Chadwick[2] using the same method, the maximum occurring for a value of H_ρ nearly twice as great as that found by him. In this connection the work of Rutherford and Robinson[3] on the fine structure of the spectrum of radium C is of interest. They found a great number of lines near $H_\rho = 4{,}000$ gauss-centimeters, in the region where our maximum was obtained. The results obtained with radium DE agree well with the photographic results of von Baeyer, Hahn, and Meitner[4] using the same source of radiations.

APPARATUS AND METHOD.

The source of the β-particles was in one experiment radium DEF chemically separated from radium, minute in quantity but very concentrated, placed on an aluminum foil and held in place by a thin aluminum leaf. In the other experiment the source was a radium salt contained in a spherical glass bulb about 5 millimeters in diameter.

A casting of lead was made of the form shown in Fig. 1, in which *O*, *O* represent the only open spaces in the casting. *S* was the position of the source, and when this was radium it was surrounded by the cylinder of

[1] The present report is presented by itself because one of us is leaving the laboratory and will continue the original plan at a later date.

[2] J. Chadwick, Verh. d. D. Phys. Ges., 16, 383, April 30, 1914.

[3] E. Rutherford and H. Robinson, Phil. Mag. (6), 26, 717, October, 1913.

[4] O. v. Baeyer, O. Hahn, and L. Meitner, Ph. ZS., 12, 378, May 15, 1911.

lead, *C*. *H* was a hemispherical counting chamber insulated by ebonite from the lead. *P* was the point connected to a string electrometer and through a high resistance to earth. The square *F*, *F*, which was 6 cm. × 6 cm., was cut out for the pole pieces of the magnet. These were 3.6 cm. apart in the apparatus used for radium. The radius of curvature of the rays was 6.34 cm. For the high values of H_ρ the same apparatus and pole pieces were used with another and stronger magnet. In the experiment with radium DEF similarly shaped apparatus was used, but this was smaller, the separation of the pole pieces being 2.5 cm. and the radius of curvature being 2.8 cm. In this smaller apparatus the counting chamber was cylindrical. A larger apparatus was needed in

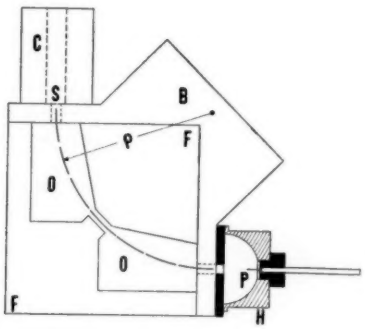

Fig. 1.

the case of radium than in the case of radium DEF because the γ-ray effect from the radium was too marked in the smaller apparatus. The increased amount of lead protecting the counting chamber was sufficient to make the γ-ray effect comfortably small with the strength of radium used (0.2 mg. radium metal).

The magnetic field strength was obtained with a Grassot fluxmeter calibrated by means of a standardized mutual inductance. The field was found approximately uniform, the greatest variation not exceeding 5 per cent. The error in H_ρ due to such non-uniformity is less than this limit.

The visual method of determining the number of β-particles has already been described by us in connection with our former researches.[1] It may suffice to state that each point was carefully standardized in a special standardizing counting chamber, both for α-particles, whose number was known from the electrical and scintillation methods, and also for β-particles from a standard radium DEF plate (prepared from emanation) used for this purpose throughout all of our researches. The

[1] A. F. Kovarik and L. W. McKeehan, PHYS. REV. (2), 6, 426, December, 1915.

voltage used was about 1,700 volts, varying somewhat for the different points. A point made of steel gave the best satisfaction. Such a point, used in counting many thousand β-particles is shown in Fig. 2, the diameter of the cylindrical part being 0.734 millimeter. The photograph was taken after the point ceased to behave properly, but whatever change caused its failure did not alter its appearance.

The hemispherical vessel, having a distribution of the field between the wall and the discharging point different from that in a cylindrical vessel, required, for the same point, a somewhat different—in this case

Fig. 2.

a higher—voltage than the cylindrical vessel, in order to give the constant maximum number of β-particles counted.

During the experiments a new method of counting was developed, but in these experiments it was used only as a check on the visual counting. The point, instead of being connected to earth through a high resistance, as is usually done, was connected to a condenser of a fairly high capacity. In this case the capacity was 0.5 microfarad. The charge collected in the condenser is due to the ions produced in the discharge to the point, and is proportional to the number of such discharges. The condenser was discharged through a ballistic galvanometer and the throw recorded. The throw per β-particle depends upon the point, and also upon the potential on the counting chamber. The throw per discharge for the α-particles, the β-particles, and for the γ-rays is characteristically different; the phenomena observed will be the subject of a later investigation. In testing the method, wires leading to the string electrometer and to earth through the high resistance were placed in proximity to the wire leading from the point to the condenser. With every discharge an induction effect produced a motion of the electrometer fiber, and this effect could be magnified by increasing greatly the high resistance leading to earth. The deflection of the fiber was, of course, in the opposite direction to that obtained when the wire was connected directly to the point. In this manner the galvanometer throw corresponding to a known number of discharges was obtained. The throw per discharge was constant within 2 per cent. for a given point, potential, and type of rays. In using this method one must frequently redetermine the throw per discharge, because this quantity changes if the point itself is becoming more or less sensitive, that is if the point's lasting qualities are poor.

RESULTS.

The results are presented in the form of curves (Figs. 3 and 4) giving the relation between H_ρ as abscissa and the relative frequency of occurrence of β-particles of the corresponding velocity as ordinate. The frequency at the maximum of each curve is arbitrarily represented by 100. The observed maximum frequencies, which depend upon the experimental dispositions and the strength of the material used, were

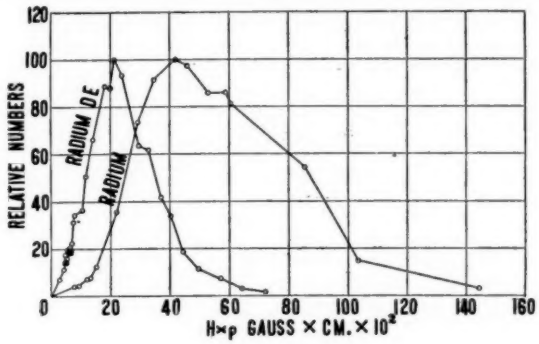

Fig. 3.

about 20 and 60 per minute for radium DE and radium with its products, respectively. The corrections necessary on account of the γ-rays and stray radiations amounted to 0.64 and 12.6 per minute in the two cases and were independent of the magnetic field strength. They were obtained by reversing the field.

Since the steps in H_ρ are in many places not sufficiently short to discover the finer details of the spectra, it is not intended, by drawing lines between the observed points, to preclude the possibility of variations in these regions. The photographic spectra already alluded to make it, in fact, practically certain that there are many such variations.

A portion of the curve for radium DE is repeated in Fig. 4 on a larger scale to show the series of maxima in the soft radiation. Besides the two maxima observed photographically by other experimenters and attributed to radium D, there are indications of two more maxima lying upon the edge of the radium E band and, perhaps for that reason, not photographically detected. The radium DE specimen was tested for the presence of radium—or rather for the radium C β-rays—by absorption experiments, and the amount present was so small that all the features of the curve are believed due to radium D and radium E. Each of the points plotted is an average of counts extending over a total time of about one hour. In order to check the working of the point without removing it from the apparatus—a procedure which may

damage a satisfactory point—a reading near the maximum was repeated at intervals throughout the experiments. This check reading was at H_ρ = 2,000 gauss-centimeters, requiring a magnetizing current of 0.46 ampere.

The curve for radium was extended in both directions as far as possible, but the extreme points are subject to large experimental errors, since the γ-ray correction mentioned above is at these points much larger

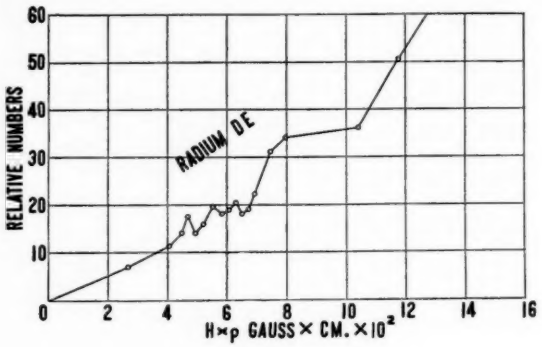

Fig. 4.

than the β-ray effect. Each of the points plotted is an average of counts extending over about half an hour. The check readings were obtained for magnetizing currents of 0.40 ampere (H_ρ = 2,190) with the smaller magnet, and 1.00 ampere (H_ρ = 4,020) with the larger magnet.

SUMMARY.

1. The magnetic spectra of the β-rays of radium DE and of radium and its products have been mapped by the statistical method.

2. The spectrum for radium DE agrees with the photographic spectra (O. v. Baeyer, O. Hahn, and L. Meitner) hitherto published. The principal maximum lies near H_ρ = 2,000 gauss-cm.

3. The spectrum for radium is different from that given by Chadwick, who also used the statistical method. The principal maximum lies near H_ρ = 4,000 gauss-cm.

4. A ballistic galvanometer method of counting is described.

In conclusion we desire to express our appreciation of the financial assistance afforded us under the provisions of the Minnesota Research Fund. We also desire to express our thanks to Mr. H. J. Vennes for his assistance throughout these experiments.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
July 20, 1916.

NEW BOOKS.

Elementos de Fisica Descritiva. By F. J. S. GOMES and A. R. MACHADO. Braga, Livraria Escolar de Cruz and Ca., 1915. Pp. 1 + 528.

This book is intended for the use of the fourth and fifth classes in the Portuguese public lyceums, which correspond to our high schools. It is attractively printed, but, although its authors are teachers in the University of Coimbra and the University of Porto respectively, it suggests a rather backward state of scientific teaching. The subject matter and the illustrations take us back at least a generation. For example, we learn that a centripetal forc is balanced by an equal centrifugal force; that in liquids the attractive and repulsive forces between molecules are equal, while in gases the latter predominate; and that intensity of radiation is proportional to the temperature of the source. The atomic theory of matter and kinetic theory of heat are referred to as established facts, but there is no discussion of the supporting evidence. On the first page of the section on light the undulatory hypothesis is mentioned, but it is completely ignored in what follows. No mention is made of interference, diffraction, or other properties of waves. The velocity of light is given, but not a hint as to any astronomical or experimental method for its determination. The section on electricity is largely devoted to electrostatics, other important branches receiving scanty mention. Mathematics is avoided; even such simple problems as those concerning image formation by lenses and mirrors are solved by purely graphical methods.

E. P. L.

Analytical Mechanics. By H. M. DADOURIAN. Second edition. New York, D. Van Nostrand Co., 1916. Pp. xiv + 431.

Some changes have been made in the second edition of this work, in order to meet the needs of engineering students. About eighty pages have been added, including a chapter on the equilibrium of framed structures and graphic statics, a list of general expressions for moments of inertia of typical bodies, a discussion of the general equations of motion of a system of particles, and a table of units and general directions for working out problems. One hundred and thirty diagrams have been added, and some three hundred practical problems. These problems have been carefully selected as illustrating fundamental principles, not merely because of their technical interest. The book is very attractively printed and has many excellent features. As in the first edition the action principle has been made the foundation of both statics and dynamics, although the statement of the principle is made in somewhat different

form. Some may question the advantages of this method of treatment. Many will likewise feel that it would have been better to omit the *ex cathedra* statement that "the new form of action which we have called kinetic reaction is the action of the ether in which the accelerated particle moves," with no suggestion of the significance of this statement, especially at this time, when our notions regarding ether, electromagnetic mass, and the structure of matter are somewhat unsettled.

E. P. L.

The Origin of the Earth. By THOMAS CHROWDER CHAMBERLIN. Chicago, The University of Chicago Press, 1916. Pp. xi + 271. Price, $1.50.

This very interesting book belongs in the middle ground between art and science (geology and physics in this instance), and shows that the driest bones of exact science may be so vitalized by the inspiring breath of artistic imagination as to serve for the framework of a living creature—even if the creature does live only in the imagination. Most attempts of this sort fail because the artist is so great an ignoramus in scientific matters that he does not even know how to get expert help on difficult points which he can not be expected to handle satisfactorily by himself; but the author of the volume before us starts from an altogether different and higher level, so that his finished work is of a correspondingly higher order of excellence.

If the number of angels who can stand on the point of one needle could be deduced by unimpeachable reasoning from plausible physical hypotheses, it might still be doubted whether the physicist would regard the result as of great importance; angels do not stand on the points of needles often enough for us to get a good chance to check up the result and so confirm or disprove the hypotheses. Similarly, it may be doubted whether the results of the author's most ingenious use of valid reasoning on plausible hypotheses will be regarded by physicists in a very serious light. But the book is highly entertaining and may be strongly recommended as vacation reading.

E. B.